DOCUMENT RESUME

ED 071 895 SE 015 532

TITLE	Project Physics Text 3, The Triumph of Mechanics.
INSTITUTION	Harvard Univ., Cambridge, Mass. Harvard Project Physics.
SPONS AGENCY	Office of Education (DHEW), Washington, D.C. Bureau of Research.
BUREAU NO	BR-5-1038
PUB DATE	68
CONTRACT	OEC-5-10-058
NOTE	169p.; Authorized Interim Version
EDRS PRICE	MF-$0.65 HC-$6.58
DESCRIPTORS	*Energy; Instructional Materials; Kinetic Molecular Theory; *Physics; Scientific Concepts; Secondary Grades; *Secondary School Science; *Textbooks; *Thermodynamics
IDENTIFIERS	Harvard Project Physics

ABSTRACT

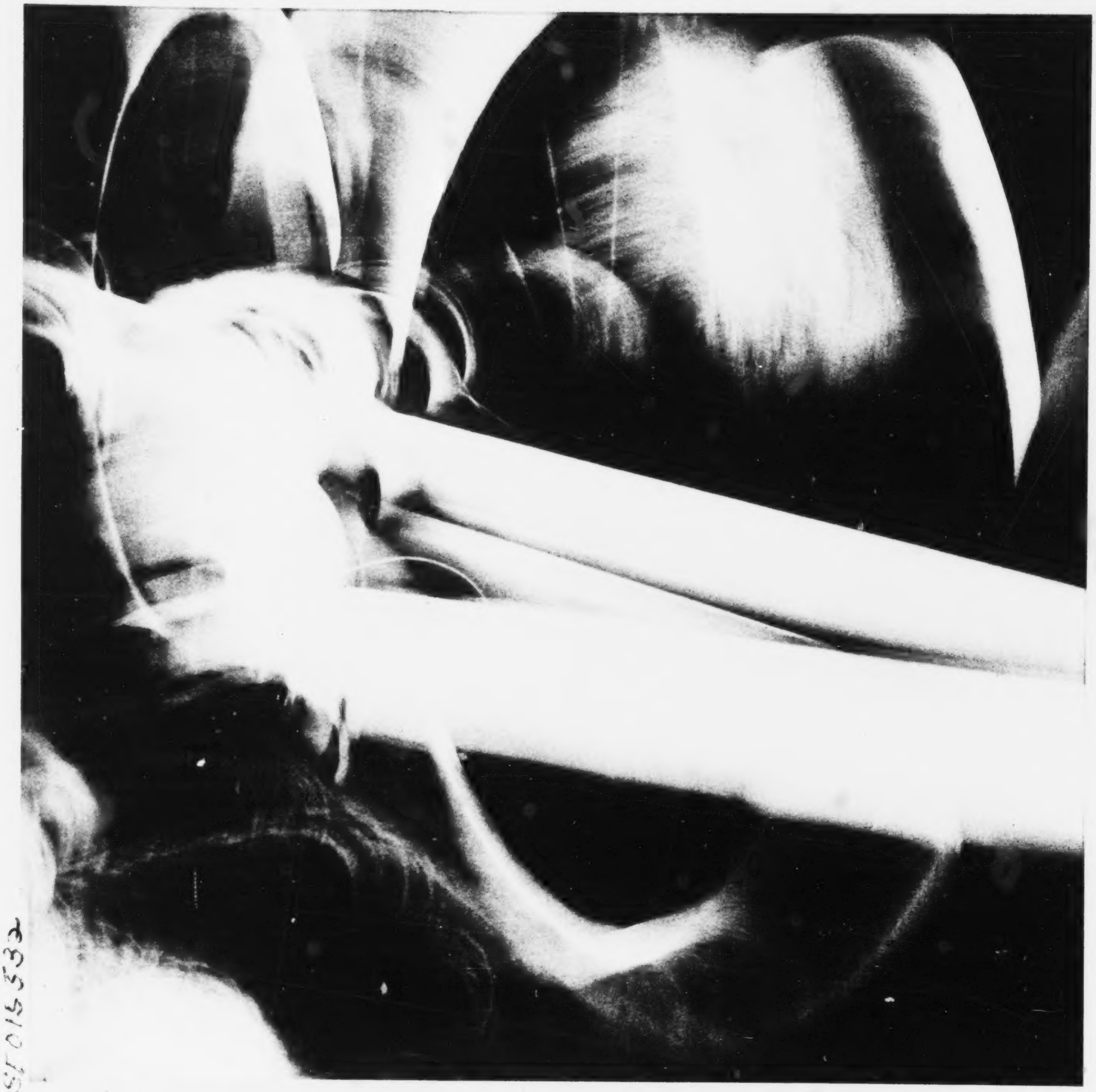
Mechanical theories are presented in this unit of the Project Physics text for senior high students. Collisions, Newton's laws, isolated systems, and Leibniz' concept are discussed, leading to conservation of mass and momentum. Energy conservation is analyzed in terms of mechanical energy, heat energy, steam engines, Watt's engine, Joule's experiment, and energy in biological systems. Kinetic theory of gases is studied in connection with molecular sizes and speeds, ideal gas, second thermodynamic law, statistical representations, time's arrow, and recurrence paradox. Wave models are introduced to deal with the superposition principle, sound properties, and wave interference, diffraction, reflection, and refraction. Historical developments are stressed in the description of this unit. Included is a chart of renowned people's life spans from 1700 to 1850. Besides illustrations for explanation use, problems with their answers are also provided in two categories: study guide and end of section questions. The work of Harvard Project Physics has been financially supported by: the Carnegie Corporation of New York, the Ford Foundation, the National Science Foundation, the Alfred P. Sloan Foundation, the United States Office of Education, and Harvard University. (CC)

ED 071895

Project Physics **Text**

An Introduction to Physics

The Triumph of Mechanics



SE 015 532

ED 071895

U.S. DEPARTMENT OF HEALTH,
EDUCATION & WELFARE
OFFICE OF EDUCATION
THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL OFFICE OF EDUCATION POSITION OR POLICY

Project Physics **Text**

An Introduction to Physics **3** The Triumph of Mechanics

Authorized Interim Version 1968-69

Distributed by **Holt, Rinehart and Winston, Inc.** New York Toronto

The Project Physics course was developed through the contributions of many people; the following is a partial list of those contributors. (The affiliations indicated are those just prior to or during association with the Project.)

90123 69 9876543

03–073445–2

Copyright © 1968, Project Physics Incorporated.

Copyright is claimed until September 1, 1968. After September 1, 1968 all portions of this work not identified herein as the subject of previous copyright shall be in the public domain. The authorized interim version of the Harvard Project Physics course is being distributed at cost by Holt, Rinehart and Winston, Inc. by arrangement with Project Physics Incorporated, a non-profit educational organization.

All persons making use of any part of these materials are requested to acknowledge the source and the financial support given to Project Physics by the agencies named below, and to include a statement that the publication of such material is not necessarily endorsed by Harvard Project Physics or any of the authors of this work.

The work of Harvard Project Physics has been financially supported by the Carnegie Corporation of New York, the Ford Foundation, the National Science Foundation, the Alfred P. Sloan Foundation, the United States Office of Education, and Harvard University.

Directors of Harvard Project Physics

Gerald Holton, Dept. of Physics, Harvard University
F. James Rutherford, Capuchino High School, San Bruno, Calif.
Fletcher G. Watson, Harvard Graduate School of Education

Advisory Committee

E. G. Begle, Stanford University, Calif.
Paul F. Brandwein, Harcourt, Brace & World, Inc., San Francisco, Calif.
Robert Brode, University of California, Berkeley
Erwin Hiebert, University of Wisconsin, Madison
Harry Kelly, North Carolina State College, Raleigh
William C. Kelly, National Research Council, Washington, D. C.
Philippe LeCorbeiller, New School for Social Research, New York, N.Y.
Thomas Miner, Garden City High School, New York, N.Y.
Philip Morrison, Massachusetts Institute of Technology, Cambridge
Ernest Nagel, Columbia University, New York, N.Y.
Leonard K. Nash, Harvard University
I. I. Rabi, Columbia University, New York, N.Y.

Staff and Consultants

Andrew Ahlgren, Maine Township High School, Park Ridge, Ill.
L. K. Akers, Oak Ridge Associated Universities, Tenn.
Roger A. Albrecht, Osage Community Schools, Iowa
David Anderson, Oberlin College, Ohio
Gary Anderson, Harvard University
Donald Armstrong, American Science Film Association, Washington, D.C.
Sam Ascher, Henry Ford High School, Detroit, Mich.
Ralph Atherton, Talawanda High School, Oxford, Ohio
Albert V. Baez, UNESCO, Paris
William G. Banick, Fulton High School, Atlanta, Ga.
Arthur Bardige, Nova High School, Fort Lauderdale, Fla.
Rolland B. Bartholomew, Henry M. Gunn High School, Palo Alto, Calif.
O. Theodor Benfey, Earlham College, Richmond, Ind.
Richard Berendzen, Harvard College Observatory
Alfred M. Bork, Reed College, Portland, Ore.
Alfred Brenner, Harvard University
Robert Bridgham, Harvard University
Richard Brinckerhoff, Phillips Exeter Academy, Exeter, N.H.
Donald Brittain, National Film Board of Canada, Montreal
Joan Bromberg, Harvard University
Vinson Bronson, Newton South High School, Newton Centre, Mass.
Stephen G. Brush, Lawrence Radiation Laboratory, University of California, Livermore
Michael Butler, CIASA Films Mundiales, S.A., Mexico
Leon Callihan, St. Mark's School of Texas, Dallas
Douglas Campbell, Harvard University
Dean R. Casperson, Harvard University
Bobby Chambers, Oak Ridge Associated Universities, Tenn.
Robert Chesley, Thacher School, Ojai, Calif.
John Christensen, Oak Ridge Associated Universities, Tenn.
Dora Clark, W. G. Enloe High School, Raleigh, N.C.
David Clarke, Browne and Nichols School, Cambridge, Mass.
Robert S. Cohen, Boston University, Mass.

Brother Columban Francis, F.S.C., Mater Christi Diocesan High School, Long Island City, N.Y.
Arthur Compton, Phillips Exeter Academy, Exeter, N.H.
David L. Cone, Los Altos High School, Calif.
William Cooley, University of Pittsburgh, Pa.
Ann Couch, Harvard University
Paul Cowan, Hardin-Simmons University, Abilene, Tex.
Charles Davis, Fairfax County School Board, Fairfax, Va.
Michael Dentamaro, Senn High School, Chicago, Ill.
Raymond Dittman, Newton High School, Mass.
Elsa Dorfman, Educational Services Inc., Watertown, Mass.
Vadim Drozin, Bucknell University, Lewisburg, Pa.
Neil F. Dunn, Burlington High School, Mass.
R. T. Ellickson, University of Oregon, Eugene
Thomas Embry, Nova High School, Fort Lauderdale, Fla.
Walter Eppenstein, Rensselaer Polytechnic Institute, Troy, N.Y.
Herman Epstein, Brandeis University, Waltham, Mass.
Thomas F. B. Ferguson, National Film Board of Canada, Montreal
Thomas von Foerster, Harvard University
Kenneth Ford, University of California, Irvine
Robert Gardner, Harvard University
Fred Geis, Jr., Harvard University
Nicholas J. Georgis, Staples High School, Westport, Conn.
H. Richard Gerfin, Simon's Rock, Great Barrington, Mass.
Owen Gingerich, Smithsonian Astrophysical Observatory, Cambridge, Mass.
Stanley Goldberg, Antioch College, Yellow Springs, Ohio
Leon Goutevenier, Paul D. Schreiber High School, Port Washington, N.Y.
Albert Gregory, Harvard University
Julie A. Goetze, Weeks Jr. High School, Newton, Mass.
Robert D. Haas, Clairemont High School, San Diego, Calif.
Walter G. Hagenbuch, Plymouth-Whitemarsh Senior High School, Plymouth Meeting, Pa.
John Harris, National Physical Laboratory of Israel, Jerusalem
Jay Hauben, Harvard University
Robert K. Henrich, Kennewick High School, Washington
Peter Heller, Brandeis University, Waltham, Mass.
Banesh Hoffmann, Queens College, Flushing, N.Y.
Elisha R. Huggins, Dartmouth College, Hanover, N.H.
Lloyd Ingraham, Grant High School, Portland, Ore.
John Jared, John Rennie High School, Pointe Claire, Quebec
Harald Jensen, Lake Forest College, Ill.
John C. Johnson, Worcester Polytechnic Institute, Mass.
Kenneth J. Jones, Harvard University
LeRoy Kallemeyn, Benson High School, Omaha, Neb.
Irving Kaplan, Massachusetts Institute of Technology, Cambridge
Benjamin Karp, South Philadelphia High School, Pa.
Robert Katz, Kansas State University, Manhattan, Kans.
Harry H. Kemp, Logan High School, Utah
Ashok Khosla, Harvard University
John Kemeny, National Film Board of Canada, Montreal
Merritt E. Kimball, Capuchino High School, San Bruno, Calif.
Walter D. Knight, University of California, Berkeley
Donald Kreuter, Brooklyn Technical High School, N.Y.
Karol A. Kunysz, Laguna Beach High School, Calif.
Douglas M. Lapp, Harvard University
Leo Lavatelli, University of Illinois, Urbana
Joan Laws, American Academy of Arts and Sciences, Boston
Alfred Leitner, Michigan State University, East Lansing
Robert B. Lillich, Solon High School, Ohio
James Lindblad, Lowell High School, Whittier, Calif.
Noel C. Little, Bowdoin College, Brunswick, Me.
Arthur L. Loeb, Ledgemont Laboratory, Lexington, Mass.
Richard T. Mara, Gettysburg College, Pa.
John McClain, University of Beirut, Lebanon
William K. Mehlbach, Wheat Ridge High School, Colo.
Priya N. Mehta, Harvard University
Glen Mervyn, West Vancouver Secondary School, B.C., Canada
Franklin Miller, Jr., Kenyon College, Gambier, Ohio
Jack C. Miller, Pomona College, Claremont, Calif.
Kent D. Miller, Claremont High School, Calif.
James A. Minstrell, Mercer Island High School, Washington
James F. Moore, Canton High School, Mass.
Robert H. Mosteller, Princeton High School, Cincinnati, Ohio
William Naison, Jamaica High School, N.Y.
Henry Nelson, Berkeley High School, Calif.
Joseph D. Novak, Purdue University, Lafayette, Ind.
Thorir Olafsson, Menntaskolinn Ad, Laugarvatni, Iceland
Jay Orear, Cornell Universilty, Ithaca, N.Y.
Paul O'Toole, Dorchester High School, Mass.
Costas Papaliolios, Harvard University
Jacques Parent, National Film Board of Canada, Montreal
Eugene A. Platten, San Diego High School, Calif.
L. Eugene Poorman, University High School, Bloomington, Ind.
Gloria Poulos, Harvard University
Herbert Priestley, Knox College, Galesburg, Ill.
Edward M. Purcell, Harvard University
Gerald M. Rees, Ann Arbor High School, Mich.
James M. Reid, J. W. Sexton High School, Lansing, Mich.
Robert Resnick, Rensselaer Polytechnic Institute, Troy, N.Y.
Paul I. Richards, Technical Operations, Inc., Burlington, Mass.
John Rigden, Eastern Nazarene College, Quincy, Mass.
Thomas J. Ritzinger, Rice Lake High School, Wisc.
Nickerson Rogers, The Loomis School, Windsor, Conn.
Sidney Rosen, University of Illinois, Urbana
John J. Rosenbaum, Livermore High School, Calif.
William Rosenfeld, Smith College, Northampton, Mass.
Arthur Rothman, State University of New York, Buffalo
Daniel Rufolo, Clairemont High School, San Diego, Calif.
Bernhard A. Sachs, Brooklyn Technical High School, N.Y.
Morton L. Schagrin, Denison University, Granville, Ohio
Rudolph Schiller, Valley High School, Las Vegas, Nev.
Myron O. Schneiderwent, Interlochen Arts Academy, Mich.
Guenter Schwarz, Florida State University, Tallahassee
Sherman D. Sheppard, Oak Ridge High School, Tenn.
William E. Shortall, Lansdowne High School, Baltimore, Md.
Devon Showley, Cypress Junior College, Calif.
William Shurcliff, Cambridge Electron Accelerator, Mass.
George I. Squibb, Harvard University
Sister M. Suzanne Kelley, O.S.B., Monte Casino High School, Tulsa, Okla.
Sister Mary Christine Martens, Convent of the Visitation, St. Paul, Minn.
Sister M. Helen St. Paul, O.S.F., The Catholic High School of Baltimore, Md.
M. Daniel Smith, Earlham College, Richmond, Ind.
Sam Standring, Santa Fe High School, Sante Fe Springs, Calif.
Albert B. Stewart, Antioch College, Yellow Springs, Ohio
Robert T. Sullivan, Burnt Hills-Ballston Lake Central School, N.Y.
Loyd S. Swenson, University of Houston, Texas
Thomas E. Thorpe, West High School, Phoenix, Ariz.
June Goodfield Toulmin, Nuffield Foundation, London, England
Stephen E. Toulmin, Nuffield Foundation, London, England
Emily H. Van Zee, Harvard University
Ann Venable, Arthur D. Little, Inc., Cambridge, Mass.
W. O. Viens, Nova High School, Fort Lauderdale, Fla.
Herbert J. Walberg, Harvard University
Eleanor Webster, Wellesley College, Mass.
Wayne W. Welch, University of Wisconsin, Madison
Richard Weller, Harvard University
Arthur Western, Melbourne High School, Fla.
Haven Whiteside, University of Maryland, College Park
R. Brady Williamson, Massachusetts Institute of Technology, Cambridge
Stephen S. Winter, State University of New York, Buffalo

Welcome to the study of physics. This volume, more of a student's guide than a text of the usual kind, is part of a whole group of materials that includes a student handbook, laboratory equipment, films, programmed instruction, readers, transparencies, and so forth. Harvard Project Physics has designed the materials to work together. They have all been tested in classes that supplied results to the Project for use in revisions of earlier versions.

The Project Physics course is the work of about 200 scientists, scholars, and teachers from all parts of the country, responding to a call by the National Science Foundation in 1963 to prepare a new introductory physics course for nationwide use. Harvard Project Physics was established in 1964, on the basis of a two-year feasibility study supported by the Carnegie Corporation. On the previous pages are the names of our colleagues who helped during the last six years in what became an extensive national curriculum development program. Some of them worked on a full-time basis for several years; others were part-time or occasional consultants, contributing to some aspect of the whole course; but all were valued and dedicated collaborators who richly earned the gratitude of everyone who cares about science and the improvement of science teaching.

Harvard Project Physics has received financial support from the Carnegie Corporation of New York, the Ford Foundation, the National Science Foundation, the Alfred P. Sloan Foundation, the United States Office of Education and Harvard University. In addition, the Project has had the essential support of several hundred participating schools throughout the United States and Canada, who used and tested the course as it went through several successive annual revisions.

The last and largest cycle of testing of all materials is now completed; the final version of the Project Physics course will be published in 1970 by Holt, Rinehart and Winston, Inc., and will incorporate the final revisions and improvements as necessary. To this end we invite our students and instructors to write to us if in practice they too discern ways of improving the course materials.

The Directors
Harvard Project Physics

An Introduction to Physics **3** The Triumph of Mechanics

Prologue 1

Chapter 9: Conservation of Mass and Momentum 8

- Conservation of mass 9
- Collisions 13
- Conservation of momentum 17
- Momentum and Newton's laws of motion 19
- Isolated systems 23
- Elastic collisions 24
- Leibniz and the conservation of *vis viva* 27

Chapter 10: Energy 36

- Work and kinetic energy 37
- Potential energy 40
- Conservation of mechanical energy 42
- Forces that do no work 44
- Heat energy and the steam engine 46
- James Watt and the Industrial Revolution 51
- The experiments of Joule 58
- Energy in biological systems 60
- The law of conservation of energy 64

Chapter 11: The Kinetic Theory of Gases

- An overview 77
- A model for the gaseous state 79
- The speeds of molecules 81
- The sizes of molecules 85
- Mathematical derivation of the ideal gas law from the kinetic theory model (special topic) 91
- The second law of thermodynamics and the dissipation of energy 98
- Maxwell's demon and the statistical view of the second law of thermodynamics 101
- Time's arrow and the recurrence paradox 103

Chapter 12: Waves 112

- Introduction 113
- Properties of waves 114
- Wave propagation 117
- Periodic waves 118
- When waves meet: the superposition principle 121
- A two-source interference pattern 123
- Standing waves 128
- Wave fronts and diffraction 131
- Reflection 134
- Refraction 138
- Sound waves 141

Epilogue 150

Index 152

Brief Answers to Study Guide 155

Answers to End of Section Questions 161

Prologue The triumph of Isaac Newton in uniting motion and astronomy is one of the glories of the human mind, a turning point in the development of science and man. Never before had a scientific theory been so successful in predicting the future, and never before had the possibilities for future development in science seemed so unlimited.

So it is not surprising that after his death in 1727 Newton was practically deified, especially in England, by poems such as this one:

> Newton the unparallel'd, whose Name
> No Time will wear out of the Book of Fame,
> Celestial Science has promoted more,
> Than all the Sages that have shone before.
> Nature compell'd his piercing Mind obeys,
> And gladly shows him all her secret Ways;
> 'Gainst Mathematics she has no defence,
> And yields t'experimental Consequence;
> His tow'ring Genius, from its certain Cause
> Ev'ry Appearance *a priori* draws
> And shews th'Almight Architect's unalter'd Laws.

(From J. T. Desagulier, *The Newtonian System of the World, the Best Model of Government, an Allegorical Poem*).

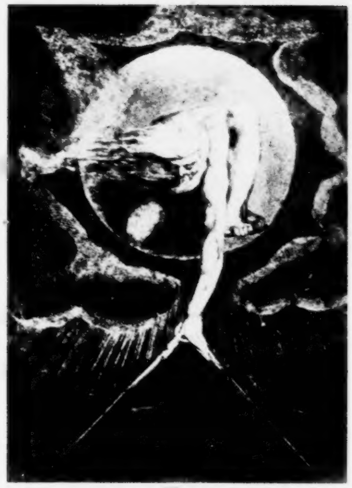
Newton's work not only led others to new heights in science, but altered profoundly man's view of the universe. Physicists after Newton explained the motion of planets around the sun by treating the solar system as a huge machine. Although the parts of the solar system are held together by gravitational forces rather than by nuts and bolts, the motion of these parts relative to each other, according to Newton's theory, is still fixed forever once the system has been put together.

We call this model of the solar system the *Newtonian world-machine*. It is a theoretical system, not a real one, because the mathematical equations which govern its motions take account of only a few of the properties of the real solar system and leave out others. In particular, the equations take no account of the structure and chemical composition of the planets, or the heat, light, electricity and magnetism which are involved. The Newtonian system takes account only of the masses, positions and velocities of the parts of the system, and the gravitational forces among them.

The idea of a world machine does not derive entirely from Newton. In his *Principles of Philosophy*, René Descartes, the most influential French philosopher of the seventeenth century, clearly stated the idea that the world is like a machine. He wrote:

> I do not recognize any difference between the machines that artisans make and the different bodies that nature alone composes, unless it be that the effects of the machines depend only upon the adjustment of certain tubes or springs, or other instruments, that, having necessarily some proportion with the hands of those who make them, are always so large that their shapes and motions can be seen, while the tubes and springs that cause the

> effects of natural bodies are ordinarily too small to be perceived by our senses. And it is certain that all the laws of Mechanics belong to Physics, so that all the things that are artificial, are at the same time natural.


"The Ancient of Days" by William Blake, an English poet who had little sympathy with the Newtonian style of "natural philosophy."

Robert Boyle (1627-1691), a British scientist who studied the properties of air (see Chapter 11), expressed the mechanistic viewpoint even in his religious writings. He argued that a God who could design a universe that would run by itself like a machine was more wonderful, and more deserving of human worship, than a God who simply created several different kinds of matter and gave each a natural tendency to behave in the way it does. Boyle also thought it was insulting to God to believe that the world-machine would be so badly designed as to require any further divine intervention after it had once been created. He suggested that an engineer's skill in designing "an elaborate engine" is more deserving of praise if the engine never needs supervision to regulate it or keep it from getting "out of order." "Just so," he continued,

> ...it more sets off the wisdom of God in the fabric of the universe, that he can make so vast a machine perform all those many things, which he designed it should, by the meer contrivance of brute matter managed by certain laws of local motion and upheld by his ordinary and general concourse, than if he employed from time to time an intelligent overseer, such as nature is fancied to be, to regulate, assist, and controul the motions of the parts....

According to Boyle and many other scientists in the seventeenth and eighteenth centuries, God was the first and greatest theoretical physicist. God, they said, set down the laws of matter and motion; human scientists can best glorify the works of God by discovering and proclaiming these laws.

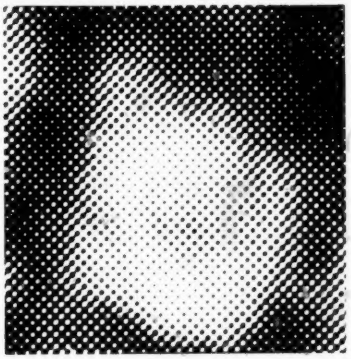
Our main concern in this unit is with physics after Newton. In mechanics the Newtonian theory was developed to accommodate a wider range of concepts. Conservation laws became increasingly important. These powerful principles offered a new way of thinking about Newtonian mechanics, and so were important in the application of Newton's theory to other areas of physics.

Newton's mechanics treats directly only a small range of experiences, those concerning the motion of simple bodies. Will the same theory work when applied to phenomena on earth as well as to those in the heavens? Are solids, liquids and gases really just machines, or mechanical systems, which can be explained by using the same ideas about matter and motion which Newton used in explaining the solar system?

At first sight, it might seem unlikely that everything can be reduced to matter and motion, because we feel, hear, smell and see many things that seem different from matter and motion. What about colors, sounds, odors, hardness and softness, temperature and so forth? Newton himself believed that the mechanical view would be useful in investigating these other properties. In the Preface to the Principia he wrote:

> I wish we could derive the rest of the phenomena of Nature by the same kind of reasoning from mechanical principles, for I am induced by many reasons to suspect that they may all depend upon certain forces by which the particles of bodies, by some causes hitherto unknown, are mutually impelled towards one another, and cohere in regular figures, or are repelled and recede from one another. These forces being unknown, philosophers have hitherto attempted the search of Nature in vain; but I hope the principles here laid down will afford some light either to this or some truer method of Philosophy.

Knowing the laws of motion, scientists after Newton strove to apply them in many different areas. We shall see in this unit how successful the Newtonians were in explaining the physical world.



A small area from the center of the picture has been enlarged to show what the picture is "really" like. Is the picture only a collection of dots? Knowing the underlying structure doesn't spoil our other reactions to the picture, but rather gives us another dimension of understanding it.

The Cultural Impact of the Newtonian Viewpoint

The century following the death of Newton in 1727 was a period of consolidation and further application of Newton's discoveries and methods. The effects of these discoveries and methods were felt also in fields outside science.

During the 1700's, the so-called Age of Reason or Century of Enlightenment, the viewpoint which we have previously called the Newtonian cosmology became firmly entrenched in European science and philosophy. The impact of Newton's achievements may be summarized thus: he had shown that man, by observing and reasoning, could uncover the workings of the physical universe. Therefore, his followers argued, man should be able to understand not only nature but also society and the human mind by the same method. As the French science writer Fontenelle (1657-1757) expressed it:

> The geometric spirit is not so bound up with geometry that it cannot be disentangled and carried into other fields. A work of morals, or politics, of criticism, perhaps even of eloquence, will be the finer, other things being equal, if it is written by the hand of a geometer.

The English philosopher John Locke (1632-1704) reinforced Newton's influence; he argued that the goal of philosophy should be to solve problems that affect our daily life, and to deal with things that we can actually know about by observation and reasoning. "Reason must be our best judge and guide in all things," he said. Locke thought that the concept of "natural law" could be used in religion as well as in physics, and the notion of a non-emotional, non-fanatical religion appealed to many Europeans who hoped to avoid a revival of the bitter religious wars of the 1600's.

Locke advanced the theory that the mind of the new-born baby contains no "innate ideas;" it is like a blank piece of paper on which anything may be written. If this is true, then it is futile to search within ourselves for a God-given sense of what is true or morally right. Instead, we must look at nature and society to discover there any "natural laws" that may exist. Conversely, if we want to improve the quality of man's mind, we must improve the society in which he lives.

Locke's view also seems to imply an "atomistic" structure of society: each person is separate from other individuals in the sense that he has no "organic" relation to them. Previously, political theories had been based on the idea of society as an organism in which each person has a prescribed place and function. Later theories based on Locke's ideas asserted that government should have no function except to protect the freedom and property of the individual person.

Although "reason" was the catchword of the eighteenth-century philosophers, we do not have to accept their own judgment that their theories about improving religion and society were necessarily the most reasonable. It might be more accurate to say that men like Voltaire approached a subject with strong opinions, and then tried to use rational arguments to justify those opinions; but these men would not give up a doctrine such as the equal rights of all men merely because they could not find a strictly mathematical or scientific proof for it. Newtonian physics, religious toleration and republican government were all advanced by the same movement, but this does not mean there is really a logical connection among them.

The dominant theme of the 1700's was moderation—the happy medium, based on toleration of different opinions, restraint of excess in any direction, and balance of opposing forces. Even reason was not allowed to ride roughshod over religious faith; atheism, which some philosophers thought to be the logical consequence of unlimited rationality, was still regarded with horror by most Europeans. The Constitution of the United States of America, with its ingenious system of "checks and balances" to prevent any faction from getting too much power, is perhaps the most enduring achievement of this period. It attempts to establish in politics a stable equilibrium of opposing forces similar to the balance between the sun's gravitational attraction and the tendency of a planet to fly off in a straight line. If the gravitational attraction increased without a corresponding increase in planetary velocity, the planet would fall into the sun; if its velocity increased without a corresponding increase in gravitational attraction, the planet would escape from the solar system, and its inhabitants would soon freeze to death. Just as Newtonian mechanics avoided the extremes of hot and cold by keeping the earth at the right distance from the sun, so the political philosophers hoped to avoid the extremes of dictatorship and anarchy by devising a system of government that was neither too strong nor too weak.

According to James Wilson (1742-1798), who played a major role in drafting the American Constitution,

> In government, the perfection of the whole depends on the balance of the parts, and the balance of the parts consists in the independent exercise of their separate powers, and, when their powers are separately exercised, then in their mutual influence and operation on one another. Each part acts and is acted upon, supports and is supported, regulates and is regulated by the rest.
>
> It might be supposed, that these powers, thus mutually checked and controlled, would remain in a state of inaction. But there is a necessity for movement in human affairs; and these powers are forced to move, though still to move in concert. They move, indeed, in a line of direction somewhat different from that, which each acting by itself would have taken; but, at the same time, in a line partaking of the natural directions of the whole—the true line of public liberty and happiness.

In literature many men welcomed the new viewpoint as a source of novel metaphors, allusions and concepts which they could use in their poems and essays. Newton's discovery that white light is composed of colors was reflected in many poems of the 1700's (see Unit 4). Samuel Johnson advocated the literary use of words drawn from the natural sciences, defining many such words in his Dictionary and illustrating their application in his "Rambler" essays.

Other writers, such as Pope and Swift, distrusted the new cosmology and so used it for purposes of satire. In his epic poem The Rape of the Lock, Pope exaggerates the new scientific vocabulary for comic effect. Swift, sending Gulliver on his travels to Laputa, describes an academy of scientists and mathematicians whose experiments and theories were as absurd as those of the Fellows of the Royal Society must have seemed to the layman of the 1700's.

The first really powerful reaction against Newtonian cosmology was the Romantic movement. This movement was started in Germany around 1780 by young writers inspired by Johann Wolfgang von Goethe. The most familiar examples of

Romanticism in English literature are the poems and novels of Blake, Coleridge, Wordsworth, Shelley, Byron and Scott.

The Romantics disliked the notion that everything should be measured by the use of numbers; they emphasized the importance of quality rather than quantity. They preferred to study the individual, unique person or experience, rather than make abstractions and generalizations. They exalted emotion and feeling over reason and calculation. They abhorred the theory that the universe is like a piece of clockwork, made of inert matter set into motion by a God who can never afterwards show His presence. As the historian and philosopher of science, E. A. Burtt said,

> ...it was of the greatest consequence for succeeding thought that now the great Newton's authority was squarely behind that view of the cosmos which saw in man a puny, irrelevant spectator (so far as a being wholly imprisoned in a dark room can be called such) of the vast mathematical system whose regular motions according to mechanical principles constituted the world of nature. The gloriously romantic universe of Dante and Milton, that set no bounds to the imagination of man as it played over space and time, had now been swept away. Space was identified with the realm of geometry, time with the continuity of number. The world that people had thought themselves living in—a world rich with colour and sound, redolent with fragrance, filled with gladness, love and beauty, speaking everywhere of purposive harmony and creative ideals—was crowded now into minute corners in the brains of scattered organic beings. The really important world outside was a world hard, cold, colourless, silent, and dead; a world of quantity, a world of mathematically computable motions in mechanical regularity. The world of qualities as immediately perceived by man became just a curious and quite minor effect of that infinite machine beyond.

The Romantics insisted that phenomena should not be analyzed and reduced to their separate parts by mechanistic explanations; instead, they argued that the whole is greater than the sum of its parts, because the whole (whether it be a single human being or the entire universe) is pervaded by a spirit that cannot be rationally explained but can only be intuitively felt.

Continental leaders of the Romantic movement, such as the German philosopher Friedrich Schelling (1775-1854), did not want to abolish scientific research. They did want to change the way research was being done, and they proposed a new type of science called "Nature Philosophy." It is important to distinguish between this nature philosophy and the older "natural philosophy," meaning just physics. The nature philosopher does not analyze phenomena into separate parts or factors which he can measure quantitatively in his laboratory—or at least that is not his primary purpose. Instead, he tries to understand the phenomenon as a whole, and looks for underlying basic principles that govern all phenomena. The Romantic philosophers in Germany regarded Goethe as their greatest scientist as well as their greatest poet, and they pointed with pride to his theory of color, which flatly contradicted Newton's theory. According to Goethe, white light does not consist of a mixture of colors; the colors are produced by the prism acting on the light which is itself pure. In the judgment of all modern physicists, Newton was right and Goethe was wrong, so nature philosophy seems to be a failure if judged by this very important example. However, this is not the whole story.

The general tendency of nature philosophy was speculative, and insofar as it encouraged speculation about ideas which could never be testable by experiment, nature philosophy was strongly condemned by scientists. Indeed, the reaction against nature philosophy was so strong that several scientific discoveries did not receive proper recognition when first announced because they were described in language "contaminated" by philosophical speculation. Among these discoveries was the generalized principle of conservation of energy, which is described in Chapter 10. It is now generally agreed by historians of science that nature philosophy played an important role in the historical origins of this discovery. This is perhaps not surprising, since the principle of conservation of energy confirms the viewpoint of nature philosophy by asserting that all the "forces of nature"—heat, gravity, electricity, magnetism and so forth—are really just different forms or manifestations of one underlying "force" which we now call energy.

It is impossible to say whether the viewpoint of nature philosophy was good or bad for science; it has often merely encouraged futile speculation and disregard for precise measurements, but in one or two very important cases it has led to important discoveries.

Much of the dislike which Romantics (and some modern artists and intellectuals) expressed for science was based on the notion that scientists claimed to be able to find a mechanistic explanation for everything, including the human mind. If everything is explained by Newtonian science, then everything is also determined. Many modern scientists no longer believe this, but scientists in the past have made statements of this kind. For example, the French mathematical physicist Laplace (1749-1827) said:

> We ought then to regard the present state of the universe as the effect of its previous state and as the cause of the one which is to follow. Given for one instant a mind which could comprehend all the forces by which nature is animated and the respective situation of the beings who compose it—a mind sufficiently vast to submit these data to analysis—it would embrace in the same formula the movements of the greatest bodies of the universe and those of the lightest atom; for it, nothing would be uncertain and the future, as the past, would be present to its eyes.

Even the ancient Roman philosopher Lucretius (100-55 B.C.), who supported the atomic theory in his poem On the Nature of Things, did not go as far as this. In order to preserve some vestige of "free will" in the universe, Lucretius suggested that the atoms might swerve randomly in their paths. This was still unsatisfactory to a Romantic scientist like Erasmus Darwin (grandfather of evolutionist Charles Darwin) who asked:

> Dull atheist, could a giddy dance
> Of atoms lawless hurl'd
> Construct so wonderful, so wise,
> So harmonised a world?

The nature philosophers thought they could discredit the Newtonian scientists by forcing them to answer this question; to say "yes," they argued, would be absurd, and to say "no" would be disloyal to their own supposed beliefs. We shall see in this Unit how successful the Newtonians were in explaining the physical world without committing themselves to any definite answer to Erasmus Darwin's question. Instead, they were led to the discovery of immensely powerful and fruitful laws of nature.

Chapter 9 Conservation of Mass and Momentum

Section		Page
9.1	Conservation of mass	9
9.2	Collisions	13
9.3	Conservation of momentum	17
9.4	Momentum and Newton's laws of motion	19
9.5	Isolated systems	23
9.6	Elastic collisions	24
9.7	Leibniz and the conservation of *vis viva*	27

SABA
RADIO
5

9.1 Conservation of mass. If the *universe is to go on forever*, then the stuff of which it is made cannot disappear. That the total amount of material in the universe does not change is really an old idea. The Roman poet Lucretius (first century B.C.) restated a belief held in Greece as early as the fifth century B.C.:

> ...and no force can change the sum of things; for there is no thing outside, either into which any kind of matter can emerge out of the universe or out of which a new supply can arise and burst into the universe and change all the nature of things and alter their motions.
>
> [On the Nature of Things]

Just twenty-two years before Newton's birth, the English philosopher Francis Bacon included the following among his basic principles of modern science:

> There is nothing more true in nature than the twin propositions that "nothing is produced from nothing" and "nothing is reduced to nothing"...the sum total of matter remains unchanged, without increase or diminution.
>
> [Novum Organon (1620), ii, 40]

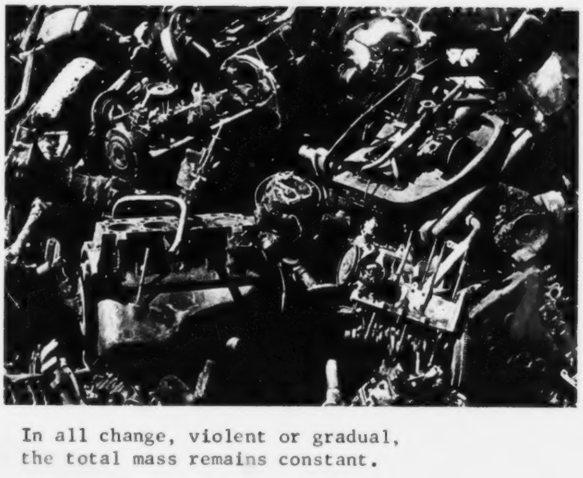
These quotations illustrate the belief that the amount of physical matter that makes up the universe remains the same—no new matter appears and no old matter disappears. While the form in which matter exists may change, matter in all our ordinary experience appears to be somehow indestructible: if we break up a large boulder into dust and pebbles, we do not change the amount of stone in the universe.

To test the belief that the quantity of matter remains constant, we need to know how to measure that quantity. Scientists recognized several centuries ago that it should not be measured by its volume. For example, if we put water in a container, mark the water level, and then freeze the water, we find that the volume of the ice is larger than the volume of the water we started with. This is true even if we are careful to seal the container so that no water can possibly come in from the outside. Similarly when we compress some gas in a closed container, the volume of the gas decreases, even though none of the gas escapes from the container.

A sample of gas undergoing a decrease in volume and increase in pressure. (This was one of the earliest high-speed flash photographs.)

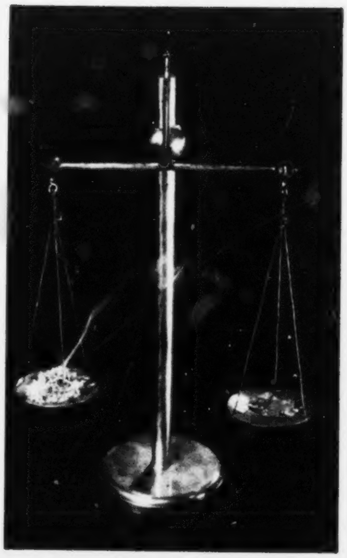
Following Newton, we have come to regard the mass of an object as the appropriate measure of the amount of matter it contains. In our use of Newtonian theory we have been assuming that the mass of an object does not change. But what if we burn the object to ashes or dissolve it in acid? Does its mass remain unchanged even in such chemical reactions?

A burnt match has a smaller mass than an unburnt one; an iron nail as it rusts increases in mass. Has the mass of these things really changed? Or does something escape from



In all change, violent or gradual, the total mass remains constant.

the match, and is something added to the iron of the nail, that will account for the changes in mass? In the eighteenth century there was a strong faith that any changes of mass in chemical reactions could be accounted for by assuming that there is something that escapes or something that enters from outside. Not until the end of the eighteenth century, however, was a sound experimental basis for this faith provided—by Antoine Lavoisier (1743-1794).



In some open-air reactions mass seems to decrease, while in others it seems to increase.

Lavoisier caused chemical reactions to occur in closed flasks and carefully weighed the flasks and their contents before and after the reaction. For example, he showed that when iron was burned in a closed flask, the mass of the iron oxide produced was equal to the sum of the masses of the iron and the oxygen used in the reaction. With experimental evidence like this at hand he was able to announce with confidence:

> We may lay it down as an incontestable axiom that in all the operations of art and nature, nothing is created; an equal quantity of matter exists both before and after the experiment,...and nothing takes place beyond changes and modifications in the combinations of these elements. Upon this principle, the whole art of performing chemical experiments depends.
>
> [Traite Elémentaire de Chimie (1789)]

Lavoisier was convinced that if he put some material in a well-sealed bottle and measured its mass, then he could return at any later time and find the same mass regardless of what happened to the material inside the bottle during the interval. Despite changes from solid to liquid or liquid to gas, etc., despite changes of color or consistency, despite even violent chemical reactions of the material inside the bottle, at least one thing would remain unchanged—the mass of what is in the bottle.

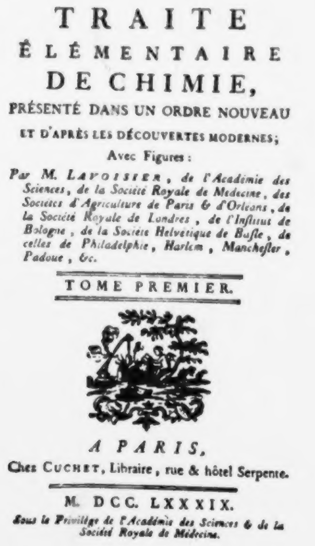
In the years after Lavoisier's pioneering work, a vast number of similar experiments were performed with ever increasing accuracy and always with the same result. As far as can be measured with sensitive balances (having an accuracy of better than 0.000001%), mass is conserved in chemical reactions, even when light and heat are allowed to enter or leave the system. Despite changes in location, shape, chemical composition and so forth, the mass of any closed system remains constant. This is the statement of what we shall call the law of conservation of mass.

The meaning of the phrase "closed system" is discussed in more detail in Sec. 9.5.

Q1 If 50 cc of alcohol is mixed with 50 cc of water, the mixture amounts to only 98 cc. Is this a contradiction of the law of conservation of mass?

Q2 It is estimated that every year at least 2000 tons of meteoric dust fall on to the earth. The dust is mostly debris

Try these end-of-section questions before going on. Answers are at the very end of the book.



TRAITE
ÉLÉMENTAIRE
DE CHIMIE,
PRÉSENTÉ DANS UN ORDRE NOUVEAU
ET D'APRÈS LES DÉCOUVERTES MODERNES;
Avec Figures :
Par M. LAVOISIER, *de l'Académie des Sciences, de la Société Royale de Medecine, des Sociétés d'Agriculture de Paris & d'Orléans, de la Société Royale de Londres, de l'Inſtitut de Bologne, de la Société Helvétique de Baſle, de celles de Philadelphie, Harlem, Mancheſter, Padoue, &c.*

TOME PREMIER.

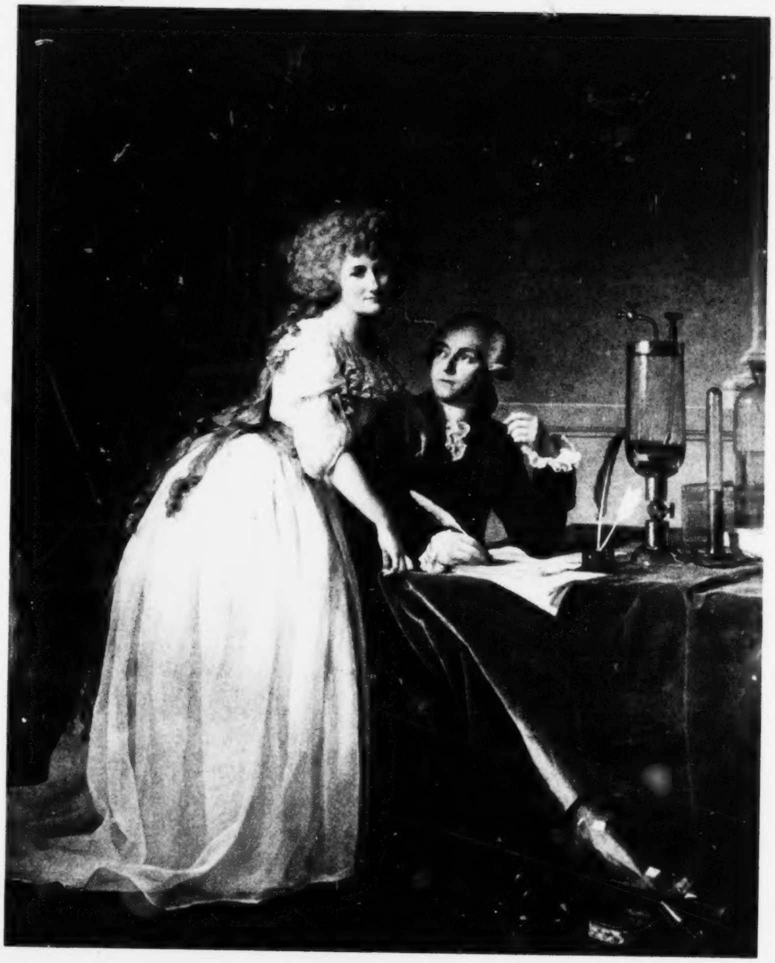
A PARIS,
Chez CUCHET, Libraire, rue & hôtel Serpente.

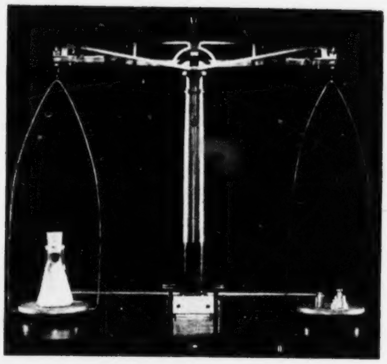
M. DCC. LXXXIX.
Sous le Privilége de l'Académie des Sciences & de la Société Royale de Médecine.

Antoine Laurent Lavoisier (1743-1794) is known as the "father of modern chemistry" because he showed the decisive importance of quantitative measurements, established the principle of conservation of mass in chemical reactions, and helped develop the present system of nomenclature for the chemical elements. He also measured the amount of heat produced by animals, and showed that organic processes such as digestion and respiration are similar to combustion.

To earn money for his scientific research, Lavoisier invested in a private company which collected taxes for the French government. Because the tax collectors were allowed to keep any extra tax which they could get from the public, they acquired a reputation for cheating and became one of the most hated groups in France. Lavoisier was not directly engaged in tax collecting, but he had married the daughter of an important executive of the company, and his association with the company was one of the reasons why Lavoisier was guillotined during the French Revolution.

Madame Lavoisier, who was only fourteen at the time of her marriage, was both beautiful and intelligent. She gave great assistance to her husband by taking notes, translating scientific works from English into French, and making illustrations. About ten years after her husband's execution, she married another scientist, Count Rumford, who is remembered for his experiments which cast doubt on the caloric theory of heat.





Conservation of mass was demonstrated in experiments on chemical reactions in closed flasks. Precision equal-arm balances were used by Lavoisier and other chemists of the time.

that was moving in orbits around the sun.

a) Is the earth a closed system as regards the law of conservation of mass?

b) How large would the system including the earth have to be in order to be considered very, very nearly closed?

c) The mass of the earth is about 6 x 10^{21} tons. Do you want to reconsider your answer to part (a)?

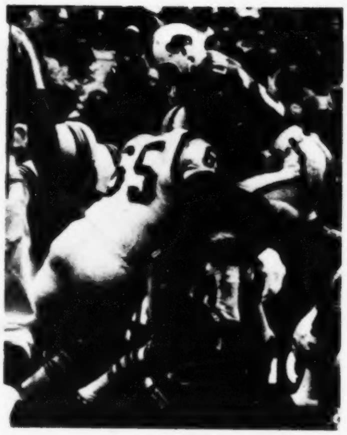
Q3 Which one of the following statements is true?

(a) Lavoisier was the first person to believe that the amount of material stuff in the universe did not change.

(b) Mass is destroyed when heat enters a system.

(c) A closed system was necessary to establish the law of conservation of mass.

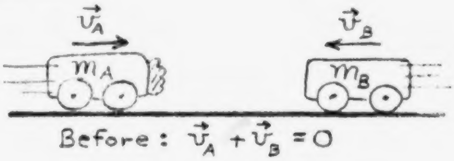
(d) The change in mass of a closed system is constant.

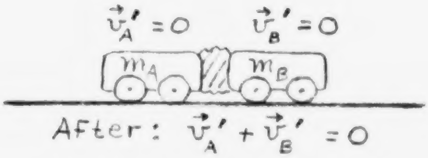


9.2 Collisions. Looking at moving things in the world around us very easily leads to the conclusion that everything set in motion eventually stops; that every clock, every machine eventually runs down. It would appear that the amount of motion in the universe is decreasing, that the universe, like any machine, is running down.

To seventeenth-century philosophers the idea of a universe that was running down was incompatible with the idea of the perfection of God; surely He would not construct such an imperfect mechanism. It was felt that if the right way could be found to define and measure motion, the quantity of motion in the universe would prove to be constant.

What is that "right" way to define and measure motion? In what way is motion sustained? What principle is at work which keeps the world machine going? To answer these questions we can look at the results of experiment. In our first experiment, we use two carts on "frictionless" wheels (or better, two dry-ice pucks or two air-track gliders) to which lumps of putty have been attached so that the carts will stick together when they collide. As you can see when you do the experiment, if the masses of the two carts are equal and if the carts are made to approach each other with equal speeds and collide head-on, they stop after the collision: their motion ceases. Is there anything related to the motion which does not change? Indeed there is. If we add the velocity $\vec{v}_A$ of one cart to the velocity $\vec{v}_B$ of the other cart, we find that the vector sum does not change. The vector sum of the velocities of the carts is zero before the collision and it is zero after the collision.



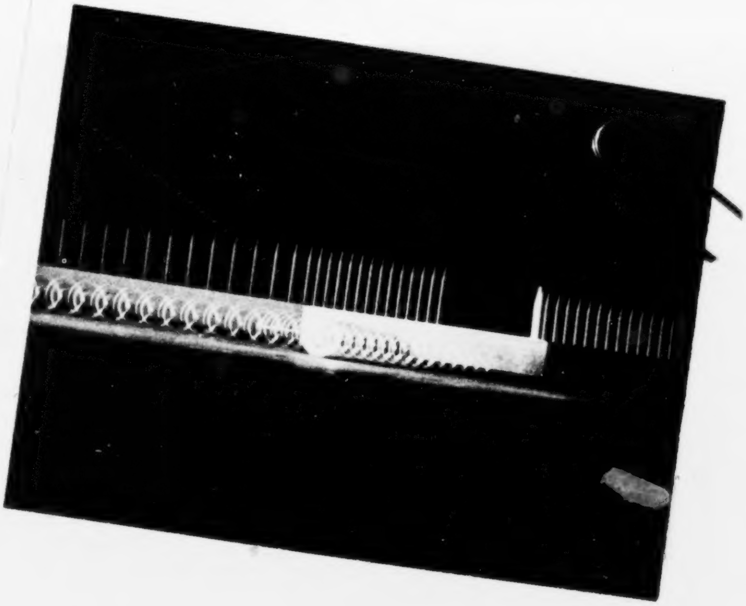


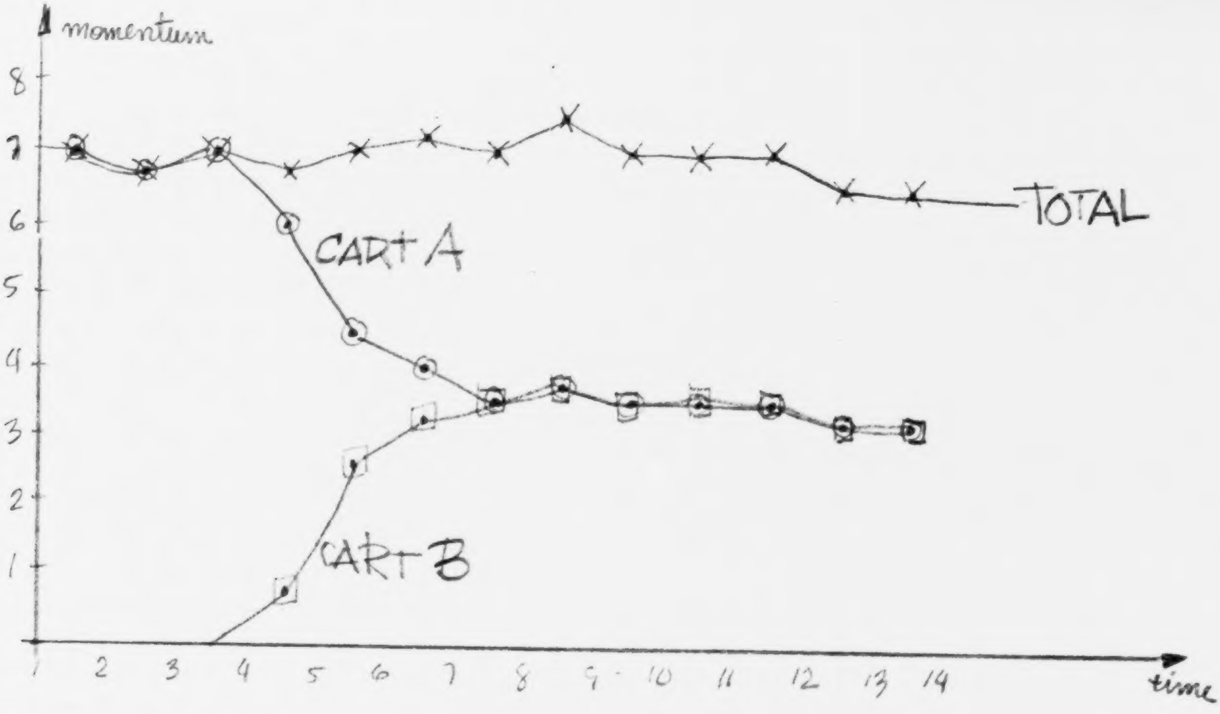
We might wonder whether this "conservation of velocity" holds for all collisions; for we have chosen

Inelastic Collision Experiment

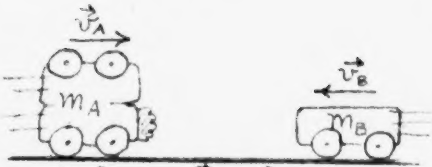
Carol Johnson
Tommy Foster
October 26

picture #	Separation cart A	cart B	Total
1			
2	7	–	7
3	6¾	–	6¾
4	7	–	7
5	6	¾	6¾
6	4½	2½	7
7	4	3¼	7¼
8	3½	3½	7
9	3¾	3¾	7½
10	3½	3½	7
11	3½	3½	7
12	3½	3½	7
13	3¼	3¼	6½
14	3¼	3¼	6½

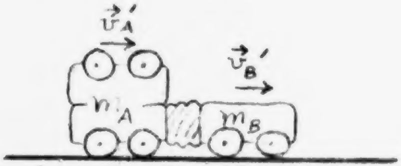




In this experiment we measured the speeds of two carts that had an inelastic collision, and saw if the total momentum was conserved. We measured speed by measuring the separtion of the pictures of the pencils on the carts. The masses of the carts were 1,057 gm and 1,063 gm. They are equal for the accuracy of our exa very special circumstance in the example above: carts with equal masses approaching each other with equal speeds. Suppose we get away from this special situation by making the mass of one of the carts twice the mass of the other cart. We can do this conveniently by putting on top of one cart another cart just like the others. Then let the carts approach each other with equal speeds and collide, as before. This time the carts do not come to rest. What happens, rather, is that there is some motion remaining, and it is in the direction of the initial velocity of the more massive cart. Our earlier guess that the vector sum of the velocities might be conserved in all collisions seems to be wrong. Another example of a collision will confirm this conclusion.

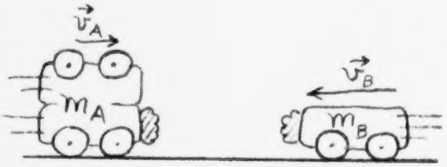


Before : $\vec{v}_A + \vec{v}_B = 0$

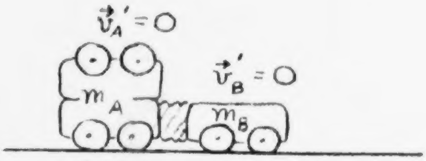


After : $\vec{v}_A' + \vec{v}_B' \neq 0$

This time let the first cart have twice as much mass as the second, and let the second cart have twice as much speed as the first. When the carts collide head-on and stick together, we find that they stop. The vector sum of the velocities is equal to zero after the collision, but is not equal to zero before the collision.



Before : $\vec{v}_A + \vec{v}_B \neq 0$



After : $\vec{v}_A' + \vec{v}_B' = 0$

It appears that the definition of quantity of motion must involve the mass of a body as well as its speed if the quantity of motion is to be the same before and after the collision. Descartes had suggested that the proper measure of a body's quantity of motion was the product of its mass and its speed. The examples above, however, show that this is not a conserved quantity. In the first and third collisions, for example, it does not equal zero for either cart before the collision, but after the collision it equals zero for each of them.

But with one very important modification of Descartes' definition of quantity of motion, we can obtain a conserved quantity. Instead of defining "quantity of motion" as the product of mass and speed, we define it, as Newton did, as the product of mass and velocity. On the next page the momentum equations are worked out for the three collisions we have considered. The conclusion is that in all three cases where we have considered head-on collisions between carts, the motion of the two carts before and after the collision is described by this equation:

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}_A' + m_B\vec{v}_B' \qquad (9.1)$$

Analyses of Three Collisions

In Section 9.2 we discussed three examples of collision between two carts. In each case the carts approached each other head-on, collided, and stuck together. We will show here that in each collision the motion of the carts before and after the collision is described by a general equation namely,

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}\,'_A + m_B\vec{v}\,'_B \qquad (9.1)$$

where m_A and m_B are the masses of the carts, and $\vec{v}_A$ and $\vec{v}_B$ are their velocities before the collision, and $\vec{v}\,'_A$ and $\vec{v}\,'_B$ are their velocities after the collision.

Example 1:

Carts with equal masses and equal speeds (but opposite directions of motion) before the collision. The speed of the stuck-together carts after the collision is zero.

In symbols:

$$m_A = m_B,$$
$$\vec{v}_B = -\vec{v}_A,$$
$$\vec{v}\,'_A = \vec{v}\,'_B = 0$$

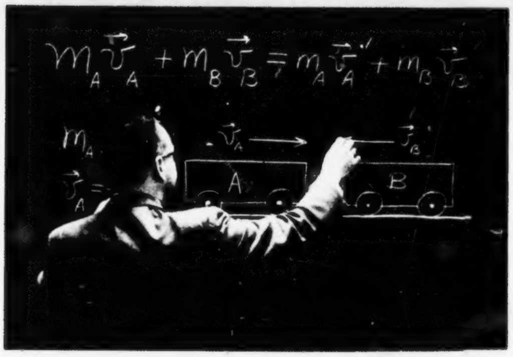
Before the collisions, the sum of the values of $m\vec{v}$ is given by:

$$\begin{aligned} m_A\vec{v}_A + m_B\vec{v}_B &= m_A\vec{v}_A + m_B(-\vec{v}_A) \\ &= m_A\vec{v}_A - m_B\vec{v}_A \\ &= (m_A - m_B)\vec{v}_A. \end{aligned}$$

Since $m_A = m_B$, the vector sum equals zero. After the collision, the sum is given by:

$$m_A\vec{v}\,'_A + m_B\vec{v}\,'_B = m_A(0) + m_B(0) = 0.$$

Thus both before and after the collision, the vector sum of the products of mass and velocity has the same value: zero. That is just what Eq. (9.1) demands; hence it does describe the collision correctly.


$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}\,'_A + m_B\vec{v}\,'_B$
m_A
$\vec{v}_A =$
$-\vec{v}_A$
$\vec{v}_B$
A
B

Example 2:

Carts with equal speeds before the collision. The mass of one cart is twice that of the other. The velocity of the stuck-together carts after the collision is found to be 1/3 the original velocity of the more massive cart.

In symbols:

$$m_A = 2m_B,$$
$$\vec{v}_B = -\vec{v}_A,$$
$$\vec{v}\,'_A = \vec{v}\,'_B = 1/3\vec{v}_A$$

Before the collision:

$$\begin{aligned} m_A\vec{v}_A + m_B\vec{v}_B &= (2m_B)\vec{v}_A + m_B(-\vec{v}_A) \\ &= m_B\vec{v}_A. \end{aligned}$$

After the collision:

$$\begin{aligned} m_A\vec{v}\,'_A + m_B\vec{v}\,'_B &= (2m_B)\frac{\vec{v}_A}{3} + m_B\frac{\vec{v}_A}{3} \\ &= m_B\vec{v}_A \end{aligned}$$

Again Eq. (9.1) describes the collision correctly, since the sum of $m\vec{v}$ is the same before and after the collision.

Example 3:

The mass of one cart is twice that of the other. The speed of the less massive truck is twice that of the more massive truck before the collision. The speed of the stuck-together carts after the collision is found to be zero.

In symbols:

$$m_A = 2m_B,$$
$$\vec{v}_B = -2\vec{v}_A,$$
$$\vec{v}\,'_A = \vec{v}\,'_B = 0$$

Before the collision:

$$\begin{aligned} m_A\vec{v}_A + m_B\vec{v}_B &= (2m_B)\vec{v}_A + m_B\,(-2\vec{v}_A) \\ &= 0 \end{aligned}$$

After the collision:

$$\begin{aligned} m_A\vec{v}\,'_A + m_B\vec{v}\,'_B &= (2m_B)\,(0) + m_B\,(0) \\ &= 0 \end{aligned}$$

Thus Eq. (9.1) can be used to describe this collision also.

where m_A and m_B are the masses of the carts, $\vec{v}_A$ and $\vec{v}_B$ are their velocities before the collision and $\vec{v}_A'$ and $\vec{v}_B'$ are their velocities after the collision.

In Unit 1, initial and final velocities were represented as $\vec{v}_i$ and $\vec{v}_f$. In Eq. (9.1) they are represented by $\vec{v}$ and $\vec{v}'$ because we now need to add subscripts such as A and B.

Q4 Why is each of the following not a good measure of quantity of motion?

a) speed
b) velocity
c) the product of mass and speed

Q5 Two carts collide head-on and stick together. In which of the following cases will the carts be at rest after the collision?

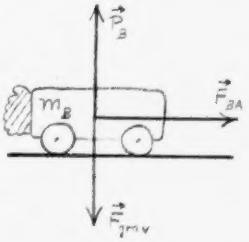
	Cart A		Cart B	
	Mass	Speed	Mass	Speed
a)	2 kg	3 m/sec	2 kg	3 m/sec
b)	2 kg	2 m/sec	3 kg	3 m/sec
c)	2 kg	3 m/sec	3 kg	2 m/sec
d)	2 kg	3 m/sec	1 kg	6 m/sec

9.3 Conservation of momentum. Equation (9.1) is the mathematical expression of a conservation law. For the colliding carts that we have considered, it says that the vector sum of the carts' quantities of motion is the same before and after the collision. Because the product of mass and velocity plays such an interesting role, we give it a better name than "quantity of motion"; we call it "momentum."

The momentum of a collection or system of objects (for example, the two carts) is the vector sum of the momenta of the objects that make up the system. In each of the collisions that we examined, the momentum of the system was the same before and after the collision. The momentum of the system was conserved.

Although we obtained it by observing collisions between two carts that stuck together when they collided, we shall see that the law of conservation of momentum is a very general law. The momentum of any system is conserved—provided that one condition is met.

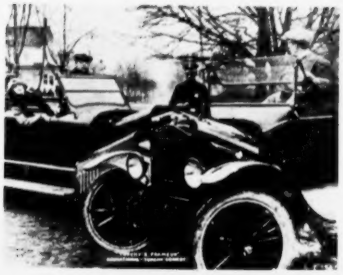
To see what that condition is, let us examine the forces acting on the carts. Each cart experiences three forces: a downward (gravitational) pull $\vec{F}_{grav}$ exerted by the earth, an upward push $\vec{P}$ exerted by the floor, and, during the collision, a push $\vec{F}$ exerted by the other cart. The first two forces evidently cancel, since the cart is not accelerating up or down. Thus the net force on each cart is just the force exerted on it by the other cart. (We are assuming that frictional forces, which are also forces exerted on the carts, have been made so small that we can neglect them here. That was the reason for using dry-ice pucks, air-track gliders or carts with "frictionless" wheels.)



Forces on cart B during collision. $\vec{F}_{BA}$ is the force exerted on cart B by cart A. (Cart A is not shown in the figure.)

9.3

The two carts comprise a system of (two) bodies. The force exerted by one cart on the other one is a force exerted by one part of the system on another part; it is not a force on the system as a whole. No net force is exerted on either cart by anything outside the system. Therefore the net force on the system as a whole is zero.

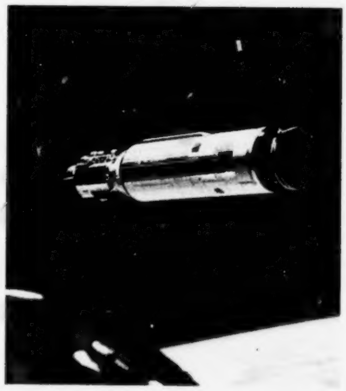


This is the condition that must be met in order for the momentum of a system of bodies to stay constant, to be conserved.

If the net force on a system of bodies is zero, the momentum of the system will not change. This is the law of conservation of momentum.

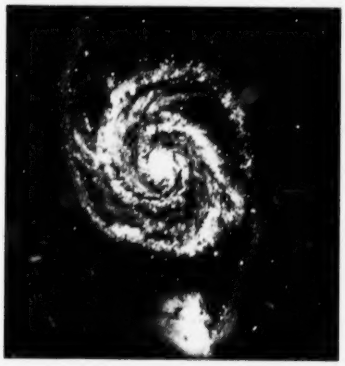
So far we have considered only instances in which two bodies exert forces on each other by direct contact and in which they stick together when they collide. But the remarkable thing about the law of conservation of momentum is how generally it applies.

It is valid no matter what kind of forces the bodies exert on each other: gravitational forces, electrical or magnetic forces, tension in strings, compression in springs, attraction or repulsion—it doesn't matter.



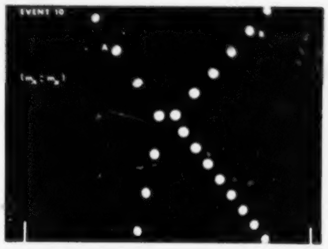
Furthermore it doesn't matter whether the bodies stick together when they collide or whether they bounce apart. They don't even have to touch (as when two strong magnets repel; see Study Guide 9.6).

The law is not restricted to systems of only two objects; there can be any number of objects in the system. Nor is the size of the system important. The law applies to the solar system as well as to an atom. The angle of the collision does not matter either. All the examples we have considered so far have been collisions between two bodies moving along the same straight line; they were "one-dimensional collisions." But if two bodies make a glancing collision rather than a head-on collision, each will move off at some angle to the line of approach. The law of conservation of momentum applies in such two-dimensional collisions also. (Remember that momentum is a vector quantity.) The law of conservation of momentum applies also in three dimensions: the vector sum of the momenta is the same before and after the collision.



On page 20 is a worked-out example to help you become familiar with the law of conservation of momentum. At the end of the chapter is a special page on the analysis of a two-dimensional collision.

There is also a variety of stroboscopic photographs and film loops of colliding bodies and exploding objects, which are available for you to analyze. They include collisions and explosions in two dimensions. Analyze as many of them as you can to see whether momentum is conserved. The more of them you analyze, the more strongly you will be convinced that the law of conservation of momentum applies to any isolated system.



One of the stroboscopic photographs that appears full-page in the Student Handbook.

In addition to illustrating the use of the law of conservation of momentum, the worked out example displays a characteristic feature of physics. Again and again, a physics problem is solved by writing down the expression of a general law (for example, $\vec{F}_{net} = m\vec{a}$) and applying it to a specific situation. Both the beginning student and a veteran research physicist find it surprising that one can do this—that a few general laws of physics enable one to solve an almost infinite number of specific individual problems. Often in everyday life people do not work from general explicit laws but rather make intuitive decisions. The way a physicist uses general laws to respond to problems can become, with practice, quite intuitive and automatic also.

All this is being said not to urge that all behavior should be deduced by applying a few general laws in every social encounter, but to explain why the process by which physicists solve problems seems a bit unfamiliar.

Q6 Which of the following has the least momentum? Which has the greatest momentum?
- a) a pitched baseball
- b) a jet plane in flight
- c) a jet plane taxiing toward the terminal

Q7 A girl on ice skates is at rest on a horizontal sheet of smooth ice. After catching a rubber ball thrown horizontally at her, she moves at 2 cm/sec. Give a rough estimate of what her speed would have been
- a) if the rubber ball were thrown twice as fast?
- b) if the rubber ball had twice the mass?
- c) if the girl had twice the mass?
- d) if the rubber ball were not caught by the girl, but bounced straight back with no change of speed?

9.4 Momentum and Newton's laws of motion. In Sec. 9.2 we discovered (perhaps "invented" is a better term) the concept of momentum and the law of conservation of momentum by considering the results of experiments with colliding carts. We did not derive the law from any theory. This is the way the law was discovered—as a generalization from experiment.

It is possible to show, however, that the law of conservation of momentum is consistent with Newton's laws of motion. In this section we shall derive the law of conservation of momentum from Newton's laws of motion.

Example of the Use of the Conservation of Momentum

Here is an example which illustrates how one can use the law of conservation of momentum.

(a) A polar bear of mass 999.9 kilograms lies sleeping on a horizontal sheet of ice. A hungry hunter fires into him a 0.1 kilogram bullet moving horizontally with a speed of 1000 m/sec. How fast does the dead bear (with the bullet imbedded in him) slide after being hit?

Mass of bullet = m_A = 0.1 kg

Mass of bear = m_B = 999.9 kg

Velocity of bullet before collision
= $\vec{v}_A$ = 1000 m/sec

Velocity of bear before collision
= $\vec{v}_B = 0$

Velocity of bullet after collision
= $\vec{v}\,'_A$

Velocity of bear after collision
= $\vec{v}\,'_B$

According to the law of conservation of momentum,

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}\,'_A + m_B\vec{v}\,'_B$$

Since $\vec{v}_B = 0$ and $\vec{v}\,'_A = \vec{v}\,'_B$, the equation becomes

$$m_A\vec{v}_A = m_A\vec{v}\,'_A + m_B\vec{v}\,'_B = (m_A + m_B)\vec{v}\,'_A.$$

$$\text{So } \vec{v}\,'_A = \frac{m_A\vec{v}_A}{(m_A + m_B)} = \frac{(0.1)(1000)\ \text{kg m/sec}}{(0.1 + 999.9)\ \text{kg}}$$

$$= \frac{(0.1)(1000)}{1000}\ \text{m/sec}$$

$$= 0.1\ \text{m/sec}.$$

The corpse of the bear slides across the ice with a speed of 10 cm/sec.

(b) Another polar bear, of mass 990 kilograms, wearing a bullet-proof vest of mass 10 kilograms, lies sleeping on the ice. The same hunter fires a bullet at him, as he did in Part (a), but the bullet bounces straight back with almost no change of speed. How fast does the bear slide after being hit by the bullet?

Mass of bullet = m_A = 0.1 kg

Mass of (bear and vest) = m_B = 1000 kg

Velocity of bullet before collision
= $\vec{v}_A$ = 1000 m/sec

Velocity of (bear and vest) before collision
= $\vec{v}_B = 0$

Velocity of bullet after collision
= $\vec{v}\,'_A$ = -1000 m/sec

Velocity of (bear and vest) after collision
= $\vec{v}\,'_B$

The law of conservation of momentum states that

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}\,'_A + m_B\vec{v}\,'_B.$$

$$(0.1)(1000) + 0 = (0.1)(-1000) + (1000)\vec{v}\,'_B$$

$$100 = -100 + 1000\ \vec{v}\,'_B$$

$$1000\ \vec{v}\,'_B = 200$$

$$\vec{v}\,'_B = 0.2\ \text{m/sec}.$$

This time the bear slides along the ice with a speed of 20 cm/sec -twice as fast as his unfortunate comrade. There is a general lesson here. It follows from the law of conservation of momentum that the struck object is given less momentum if it absorbs the bullet than if it reflects it.

9.4

Newton's second law is a relation between the net force acting on a body and the mass of the body and its acceleration: $\vec{F}_{net} = m\vec{a}$. We can also write the law in terms of the momentum of the body. If we remember that acceleration is the rate-of-change of velocity, we can write the second law as

$$\vec{F}_{net} = m\,\frac{\Delta\vec{v}}{\Delta t} \qquad (9.3)$$

or

$$\vec{F}_{net}\,\Delta t = m\,\Delta\vec{v}. \qquad (9.4)$$

If the mass of the body is constant, the change in its momentum $\Delta(m\vec{v})$ is the same as its mass times its change in velocity $m(\Delta\vec{v})$. Then we can write Eq. (9.4) as

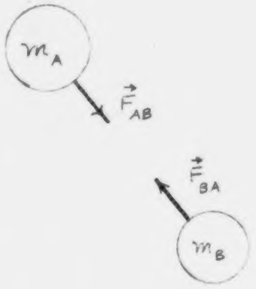
If m is a constant,

$$\begin{aligned}\Delta(m\vec{v}) &= m\vec{v}' - m\vec{v}\\ &= m(\vec{v}' - \vec{v})\\ &= m\Delta\vec{v}.\end{aligned}$$

$$\vec{F}_{net}\,\Delta t = \Delta(m\vec{v}), \qquad (9.5)$$

that is, the product of the net force on a body and the time during which this force acts equals the change in momentum of the body.

Newton's second law, in the form of Eq. (9.5), together with Newton's third law, is enough to enable us to derive the law of conservation of momentum.



Suppose two bodies with masses m_A and m_B exert forces on each other. $\vec{F}_{AB}$ is the force exerted on body A by body B and $\vec{F}_{BA}$ is the force exerted on body B by body A. No other unbalanced force acts on either body. By Newton's third law the forces $\vec{F}_{AB}$ and $\vec{F}_{BA}$ are equal in magnitude and opposite in direction. Each body acts on the other for exactly the same time Δt. Newton's second law, Eq. (9.5), applied to each of the bodies, says

$$\vec{F}_{AB}\,\Delta t = \Delta(m_A\vec{v}_A) \qquad (9.6a)$$

$$\vec{F}_{BA}\,\Delta t = \Delta(m_B\vec{v}_B). \qquad (9.6b)$$

But $\vec{F}_{AB} = -\vec{F}_{BA}$, so the left side of Eq. (9.6a) and the left side of Eq. (9.6b) are equal in magnitude and opposite in direction. Therefore the same must be true of the right sides; that is,

$$\Delta(m_A\vec{v}_A) = -\Delta(m_B\vec{v}_B). \qquad (9.7)$$

Suppose that the masses m_A and m_B are constant. Let $\vec{v}_A$ and $\vec{v}_B$ stand for the velocities of the two bodies at some instant and let $\vec{v}_A{}'$ and $\vec{v}_B{}'$ stand for their velocities at some later instant. Then Eq. (9.7) becomes

$$m_A\vec{v}_A{}' - m_A\vec{v}_A = -(m_B\vec{v}_B{}' - m_B\vec{v}_B). \qquad (9.8)$$

A little rearrangement of Eq. (9.8) leads to

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}_A{}' + m_B\vec{v}_B{}',$$

As always with equations containing vector qualities, the + and - signs indicate vector addition and vector subtraction.

Perhaps you could see even from Eq. (9.7) that conservation of momentum is implied. The change of momentum of one body is accompanied by an equal and opposite change of momentum of the other body. Accordingly the momentum of the system does not change during the time interval Δt.

which we recognize as Eq. (9.1), the law of conservation of momentum. The vector sum of the momenta of the bodies at any instant is the same as the vector sum of the momenta of the bodies after a time interval Δt, provided that during that time interval the only unbalanced force acting on either body is the force exerted on it by the other body.

What we have done here with a system consisting of two bodies, we could do as well with a system consisting of any number of bodies. Study Guide 9.18 shows you how to derive the law of conservation of momentum for a system of three bodies.

Actually Eq. (9.5) is more general than its derivation indicates. We considered a body with constant mass, but in Eq. (9.5) the change of momentum can arise from a change of mass as well as from (or in addition to) a change of velocity.

Are there any situations where the mass of a body changes? As a rocket spews out exhaust gases, its mass decreases. The mass of a train of coal cars increases as it moves under a hopper which drops coal into the cars. In Unit 5 you will learn that any body's mass increases as it moves faster and faster (although noticeably so only at extremely high speeds).

Since $\vec{F}_{net} = m\vec{a}$ expresses Newton's second law for cases where the mass is constant, it may not be so easy to use this form to deal with situations, like those above, where the mass changes; we must use the law in the form of Eq. (9.5). In fact, that is more nearly the way that Newton formulated the law in his Principia.

We have shown that the law of conservation of momentum can be derived from Newton's second and third laws. The law of conservation of momentum is not a new principle of physics but is already implicit in Newton's laws. Nevertheless the law of conservation of momentum enables us to solve many problems which would be impossible or difficult to solve using Newton's laws.

For example, suppose a cannon fires a shell. Although initially at rest, the cannon moves after firing the shell; it recoils. The expanding gases in the cannon barrel push the cannon backward just as hard as they push the shell forward. If we knew the magnitude of the force, we could apply Newton's second law to the cannon and to the shell to find their accelerations, and after a few more steps we could find the speed of the shell and the recoil speed of the cannon. But, in fact, we do not know the magnitude of the force; it is probably not steady and it almost certainly decreases as the shell moves toward the end of the barrel. Hence it would be very difficult to use Newton's laws.

We can use the law of conservation of momentum, however, even if we know nothing about the force (except that the force on the shell is always the same magnitude as the force on the cannon). It tells us that since the momentum of the system (cannon-plus-shell) is zero initially, it will also be zero after the shell is fired. Thus the law of conservation of momentum can be applied to cases where we do not have enough information to apply Newton's laws.

We pay for our ignorance, however. If we could use Newton's laws we would be able to calculate the speed of the shell and the speed of the cannon. But the law of conservation of momentum only makes it possible to calculate the relative speeds of shell and cannon.

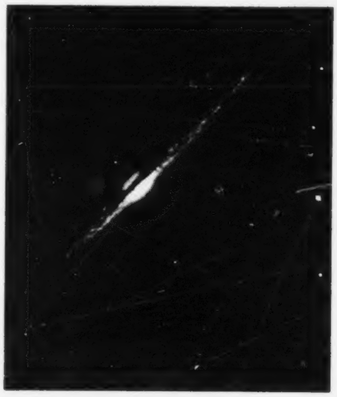
Mathematically, the reason we obtain more information when we use Newton's laws is that in that case we have two equations for the two unknown speeds. The law of conservation of momentum provides only one equation for the two unknown speeds.

Q8 a) The five engines of the first stage of the Apollo/Saturn rocket together develop a thrust of 35 million newtons for 150 seconds. How much momentum will they give the rocket?
b) The final speed of the rocket is 6100 miles/hour. Why can you not compute its mass?

Q9 Newton's second law can be written $\vec{F}\Delta t = \Delta(m\vec{v})$. Use the second law to explain the following:
a) When jumping down from a chair, you should bend your knees.
b) Firemen use elastic nets to catch people who jump out of burning buildings.
c) Hammer heads are generally made of steel rather than rubber.
d) The 1968 Pontiac GTO has plastic bumpers which, when deformed, slowly return to their original shape.

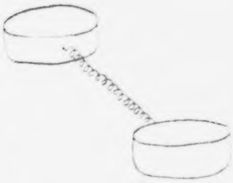
Q10 A girl is coasting down a hill on a skate board. Can she make a sharp turn without touching the ground with her foot?

9.5 Isolated systems. There are similarities in the ways we test and use the conservation laws of mass and linear momentum. In both cases we test the laws by observing systems that have in some sense been isolated or closed off from the rest of the universe.



When testing or using the law of conservation of mass we arrange a system that is closed, so that matter can neither enter nor leave. When testing or using the law of conservation of momentum we set up a system which is isolated in the sense that each body in the system experiences no net force originating from outside the system.

It may be difficult or even impossible actually to isolate a system completely. Consider for example two dry-ice pucks



connected by a spring and sliding on a frictionless table. The pucks and spring form a system from which we exclude the table and the earth, whose effects on each puck cancel. For although each puck experiences a downward gravitational force exerted by the earth, the table exerts an equally strong upward push on it. Thus it appears that the net force exerted on each puck by bodies outside the system is zero.

But what about the gravitational attraction of the sun or the moon? Or electric or magnetic forces exerted on the pucks by bodies outside the system? We cannot be sure that all such forces are completely balanced, and therefore the spring and two pucks do not actually form a completely isolated system. Nevertheless, for all real cases, the unbalanced forces usually can be kept negligibly small compared to the forces exerted by the bodies in the system on one another, so that for all practical purposes, the system can be considered isolated. For example, if two cars are skidding toward a collision, the frictional force exerted by the road on each car may be large, perhaps a few hundred pounds. Nevertheless, it may be considered negligible compared to the immense force (many tons) exerted by each car on the other when they collide; during the collision we can consider the two skidding cars as very nearly an isolated system. If friction is too great to ignore, the law of conservation of momentum still holds, but we must apply it to a system which includes the objects that provide the friction. In the case of the skidding cars, we would have to include the entire earth in the "closed system".

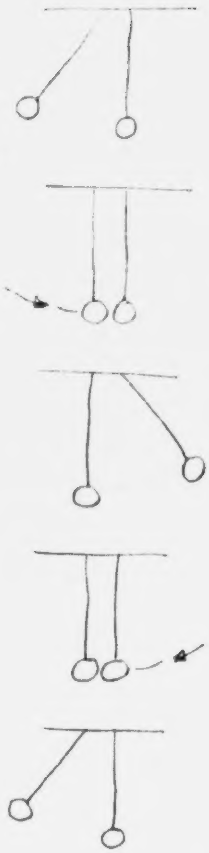
Q11 Which of the following is the way that we define an isolated system when using the law of conservation of momentum?
- a) a system in which each object experiences a net force of zero
- b) a system in which each object experiences no forces exerted by objects outside the system
- c) a system on which the net force is zero
- d) a system in which the objects exert no forces on one another

Q12 Explain why the following are not isolated systems.
- a) a baseball thrown horizontally
- b) an artificial earth satellite
- c) the earth and the moon

9.6 Elastic collisions. In 1666, members of the recently-formed Royal Society of London witnessed an experiment at one of the Society's regular meetings.

Two balls made of the same hard wood and of equal size were suspended at the ends of two strings to form two pendulums. When one ball was released from rest at a certain height, it swung down and struck the other, which had been hanging without moving.

After impact the second ball swung up to nearly the same height as that from which the first had been released and the first became very nearly motionless. When the second ball returned and struck the first, it was now the second ball which came nearly to rest, and the first swung up to almost the same height as that from which it had originally been released. And so the motion continued, back and forth through many swings.



This demonstration aroused considerable interest among members of the Society. In the years immediately following it also generated heated and often confusing arguments. Why did the balls rise each time to nearly the same height? Why was the motion transferred from one ball to the other when they collided? Why didn't the first ball bounce back or continue moving slowly forward? Momentum conservation doesn't prohibit such behavior.

In 1668 the membership directed its secretary to write to three men who could be expected to throw light on the whole matter of impacts. The three men were John Wallis, Christopher Wren and Christiaan Huygens. Within a few months, all three men replied. Wallis and Wren had partial answers to explain some of the features of collisions, but Huygens analyzed the problem in complete detail.

His work was performed in 1667, but his reasoning did not become public until 1703, when his works, including "On the Movement of Bodies through Impact," were published posthumously.

Huygens explained the behavior of the pendulums by showing that in such collisions another conservation law is at work, in addition to the law of conservation of momentum. Specifically, he showed that

$$m_A v_A^2 + m_B v_B^2 = m_A v_A'^2 + m_B v_B'^2. \qquad (9.9)$$

The quantity mv^2 came to be called *vis viva*. Eq. (9.9), then, is the mathematical expression of the conservation of *vis viva*.

Vis viva is Latin for "living force." Seventeenth century scientists were greatly interested in distinguishing and naming various "forces." They used the word loosely; it meant sometimes a push or a pull (the modern use of the word), sometimes what we now call "momentum" and sometimes what we now call "energy."

Scientists no longer use the term.

We have seen that the momentum of every isolated system of bodies is conserved, no matter what goes on within the system. Is the law of conservation of *vis viva* equally general? Is the *vis viva* of every isolated system conserved? It is easy to see that it is not.

Consider the first example of Sec. 9.2, in which two carts of equal mass approach each other with equal speeds, collide, stick together and stop. The *vis viva* of the system after the collision is

$$m_A v_A'^2 + m_B v_B'^2 = m_A(0) + m_B(0) = 0.$$

Before the collision the *vis viva* of the system was $m_A v_A^2 + m_B v_B^2$. Since $m_A v_A^2$ and $m_B v_B^2$ are both positive numbers, their sum cannot possibly equal zero (unless both v_A and v_B are zero, in which case there would be no collision—and not much of a problem). *Vis viva* is not conserved in this collision.

The law of conservation of *vis viva*, then, is not as general as the law of conservation of momentum. If two bodies collide, the *vis viva* may or may not be conserved.

As Huygens pointed out, it is conserved if the colliding bodies are hard so that they do not crumple or smash or dent. We call such bodies "perfectly elastic," and we describe collisions between them as "perfectly elastic collisions." In perfectly elastic collisions both momentum and *vis viva* are conserved.

In most collisions that we witness, *vis viva* is not conserved; the total *vis viva* after the collision is less than before the collision. Such collisions might be called almost perfectly elastic, or partially elastic, or perfectly inelastic, depending on how much of the *vis viva* is "lost" in the collision. The loss of *vis viva* is greatest when the colliding bodies remain together.

A collision between steel ball-bearings or glass marbles is almost perfectly elastic, provided that the collision is not so violent as to damage the bodies; the total *vis viva* after the collision might be as much as, say, 98% of its value before the collision. Examples of true perfectly elastic collisions are found only in collisions between atoms or between sub-atomic particles.

Q13 *Vis viva* is conserved
- a) in all collisions.
- b) only when momentum is not conserved.
- c) in some collisions.
- d) when the colliding objects are not too hard.
- e) only in living systems.

Q14 True or false:
Huygens demonstrated an experiment to the members of the Royal Society that aroused much interest.

Q15 In the diagram of the pendula on the last page, why are the strings parallel instead of attached to the same point at the ceiling?

Q16 *Vis viva* is never negative because
- a) it is impossible to draw vectors with negative length.
- b) speed is always greater than zero.
- c) it is proportional to the square of the speed.
- d) scalar quantities are always positive.

9.7 Leibniz and the conservation of *vis viva*. Descartes, believing that the total quantity of motion in the universe did not change, proposed to define the quantity of motion of an object as the product mv of its mass and its speed. As we have seen however, this is not a conserved quantity in all collisions. For example, if two carts with equal masses and equal speeds collide head-on and stick together, they both stop after the collision.

"It is wholly rational to assume that God, since in the creation of matter he imparted different motions to its parts, and preserves all matter in the same way and conditions in which he created it, so he similarly preserves in it the same quantity of motion."

[Descartes, Principles of Philosophy, 1644]

Gottfried Wilhelm Leibniz was aware of the error in Descartes' work on motion. In a letter in 1680 he wrote

> M. Descartes' physics has a great defect; it is that his rules of motion or laws of nature, which are to serve as the basis, are for the most part false. This is demonstrated. And his great principle, that the same quantity of motion is conserved in the world, is an error.

Leibniz' philosophical studies made him sure that something was conserved, however; namely, what he called "force". He described what he meant by "force" in a paper published in 1686 entitled, "A short demonstration of a famous error of Descartes and other learned men, concerning the claimed law of nature according to which God always preserves the same quantity of motion; by which, however, the science of mechanics is totally perverted." He said that

> the force must be evaluated by the quantity of effect it can produce, for example by the height to which it can raise a heavy load... and not by the velocity it can impress on [the load].

In this way he was led to identify "force" as *vis viva* mv^2. As he stated in his "Essay in Dynamics on the Laws of Motion," written in 1691,

> Now it is found from reasoning and experience that it is the absolute *vis viva* or the force measured by the violent effect it can produce which is conserved and not at all the quantity of motion.

If an object is thrown upward, the height y to which it will rise is proportional to the square of its initial speed v:

$$y = \frac{1}{2}v^2/a_g.$$

Leibniz believed that even at the top of its trajectory, a stone that has been thrown upwards still possesses "force"; as it falls back toward the ground, gaining speed, the "force" becomes evident again as *vis viva*.

But, as Huygens had pointed out, *vis viva* is conserved in collisions only if the colliding bodies are hard, that is, only in elastic collisions. In inelastic collisions the total *vis viva* after the collision is less than before the collision. It seems that Leibniz' *vis viva* is no more a conserved quantity than Descartes' quantity of motion.

For philosophical reasons, however, Leibniz was convinced that *vis viva* is always conserved. In order to save his conservation law, he was obliged to invent an ingenious but unproven explanation for its apparent disappearance in inelastic collisions. He maintained that in such collisions the *vis viva* is not destroyed "but dissipated among the small parts" of which the colliding bodies are made.

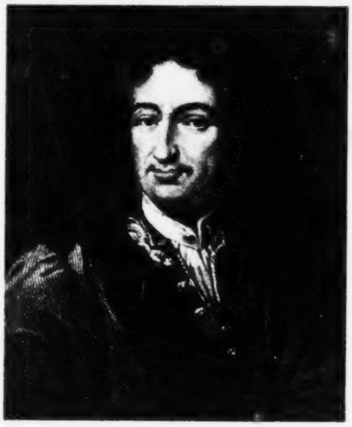
By "small parts" Leibniz did not mean atoms. His contemporaries were speculating about the existence of atoms, and he himself had accepted the idea in his youth, but "reason made me change this opinion."

The question whether quantity of motion mv or *vis viva* mv^2 was the "correct" conserved quantity was much debated by philosophers and scientists in the early eighteenth century. In fact, each theory developed into concepts which are fundamental to modern science. Descartes' mv developed into the modern $m\vec{v}$, which, as we have seen in Sec. 9.3 of this chapter, is always conserved in an isolated system. Leibniz' *vis viva* mv^2 very closely resembles the modern concept of energy of motion, kinetic energy, as we shall see in Chapter 10. Energy, of which energy of motion is only one of many forms, is also a conserved quantity in an isolated system.

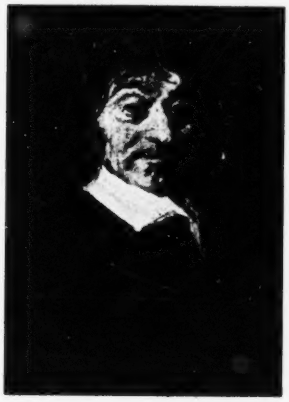
Q17 According to Leibniz, Descartes' principle of conservation of quantity of motion was

a) correct, but trivial.
b) another way of expressing the conservation of vis viva.
c) incorrect.
d) correct only in elastic collisions.

Q18 How did Leibniz explain the apparent disappearance of *vis viva* in inelastic collisions?



Leibniz (1646-1716), a contemporary of Newton, was a German philosopher and diplomat, an adviser to Louis XIV of France and Peter the Great of Russia. Independently of Newton he invented the method of mathematical analysis called calculus. A long public dispute resulted between the two great men concerning charges of plagiarism of ideas.



Descartes (1596-1650) was the most important French scientist of the seventeenth century. In addition to his early contribution to the idea of momentum conservation, he is remembered by scientists as the inventor of coordinate systems and the graphical representation of algebraic equations. His system of philosophy, which used the deductive structure of geometry as its model, is still influential. Of a sickly constitution, he did much of his thinking while lying in bed.

John Wallis (1616-1703) was a professor of mathematics at Oxford. He was later to make one of the earliest careful analyses of vibrations, thereby discovering how partial tones are produced on a stringed instrument. He also introduced the symbol ∞ for infinity.

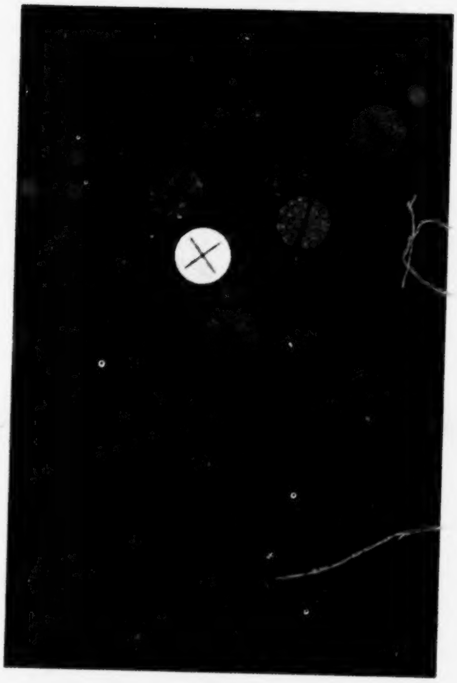
St. Paul's Cathedral

Christiaan Huygens (1629-1695) was a Dutch physicist. He devised an improved telescope with which he discovered a satellite of Saturn and saw Saturn's rings clearly. He was the first to understand centripetal force, and he invented the pendulum-controlled clock. We shall hear more of Huygens later. Huygens' scientific contributions were major, and his reputation would undoubtedly have been even greater had he not been overshadowed by his contemporary, Newton.

Christopher Wren (1632-1723) was a professor of astronomy at Oxford and was generally considered to be one of the outstanding mathematicians of the day. He is best remembered as the imaginative architect of many impressive buildings in London, including more than fifty churches and cathedrals—Saint Paul's among them.

A Collision in Two Dimensions

The stroboscopic photograph shows a collision between two wooden discs. The discs are on tiny plastic spheres which make their motion nearly frictionless. Body B (marked +) is at rest before the collision. After the collision it moves to the left and Body A (marked -) moves to the right. The mass of Body B is twice the mass of Body A: $m_B = 2m_A$. We will analyze the photograph to see whether momentum was conserved. (Note: the size reduction factor of the photograph and the (constant) stroboscopic flash rate are not given. So long as all velocities are measured in the same units, it does not matter what those units are.)



In this analysis we will measure the distance the discs moved on the photograph, in centimeters. We will use the time between flashes as the unit of time. Before the collision, Body A traveled 36.7 mm in the time between flashes: v_A = 36.7 speed-units. Similarly v'_A = 17.2 speed-units and v'_B = 11.0 speed-units.

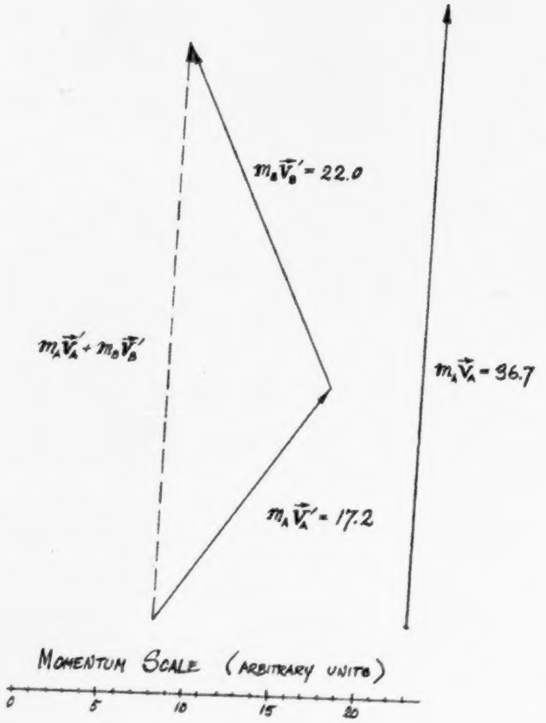
The vector diagram shows the momenta $m_A\vec{v}'_A$ and $m_B\vec{v}'_B$ after the collision; $m_A\vec{v}'_A$ is represented by a vector 17.2 momentum-units long, and $m_B\vec{v}'_B$ by a vector 22.0 momentum-units long. (Remember that $m_B = 2m_A$.)

The dotted line represents the vector sum of $m_A\vec{v}'_A$ and $m_B\vec{v}'_B$; i.e., the total momentum after the collision. Measurement shows it to be 34.0 momentum-units long.

The total momentum before the collision is just $m_A\vec{v}_A$ and is represented by a vector 36.7 momentum-units long. Thus, our measured values of the total momentum before and after the collision differ by only 2.7 momentum-units, a difference of about 7%.

You can verify for yourself that the direction of the total momentum is the same before and after the collision.

Have we then demonstrated that momentum was conserved in the collision? Is the 7% difference likely to be due entirely to experimental inaccuracies, or is there any reason to expect that the total momentum of the two discs after the collision is really a bit less than before the collision?



Study Guide

9.1 Discuss the law of conservation of mass as it applies to the following situations.

- a) a satellite "soft-landing" on Venus
- b) a rifle firing a bullet
- c) the manufacture of styrofoam
- d) a person drinking a Coke

9.2 Would you expect that in your lifetime, when more accurate balances are built, you will see experiments which show that the law of conservation of mass is not entirely true for chemical reactions?

9.3 Dayton C. Miller was a renowned experimenter at Case Institute of Technology. He was able to show that two objects placed side by side on a pan balance did not balance two identical objects placed one on top of the other. The reason is that the pull of gravity decreases with distance from the center of the earth. Would Lavoisier have said this experiment contradicted the law of conservation of mass?

9.4 A children's toy known as a snake consists of a tiny pill of mercuric thiocyanate. When the pill is ignited, a large, serpent-like foam curls out almost from nothingness. Devise and describe an experiment by which you could test the law of conservation of mass for this demonstration.

9.5 Consider the following chemical reaction, which was studied by Landolt in his tests of the law of conservation of mass. A solution of 19.4 g of potassium chromate in 100.0 g of water is mixed with a solution of 33.1 g of lead nitrate in 100.0 g of water. A bright yellow solid precipitate forms and settles to the bottom of the container. When removed from the liquid, this solid is found to have a mass of 32.3 g and is found to have properties different from either of the reactants.

- a) What is the mass of the remaining liquid? (Assume the combined mass of all substances in the system is conserved.)
- b) If the remaining liquid (after removal of the yellow precipitate) is then heated to 95° C, the water it contains will evaporate, leaving a white solid. What is the mass of this solid? [Assume that the water does not react with anything, either in (a) or in (b).]

9.6 If powerful magnets are placed on top of each of two carts, and the magnets are so arranged that like poles face each other as one cart is pushed toward the other, the carts bounce away from each other without actually making contact.

- a) In what sense can this be called a collision?
- b) Does the law of conservation of momentum apply?
- c) Does the law of conservation of momentum hold for any two times during the interval when the cars are approaching or receding (the first called before and the second called after the "collision")?

9.7 A freight car of mass 10^5 kg travels at 2.0 m/sec and collides with a motionless freight car of mass 1.5×10^5 kg. The two cars lock and roll together after impact. Find the final velocity of the two cars after collision.

HINTS:

- a) Equation (9.1) states

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}\,'_A + m_B\vec{v}\,'_B$$

  What factors in this equation are given in the problem?
- b) Rearrange terms to get an expression for $\vec{v}\,'_A$.
- c) Find the value of $\vec{v}\,'_A$ ($\vec{v}\,'_A = \vec{v}\,'_B$).

9.8 From the equation

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}\,'_A + m_B\vec{v}\,'_B$$

show that the change in momentum of object A is equal and opposite to the change of momentum of object B.

9.9 Benjamin Franklin, in correspondence with his friend James Bowdoin (founder and first president of the American Academy of Arts and Sciences), objected to the corpuscular theory of light by saying that a particle traveling with such immense speed (3×10^8 m/sec) would have the impact of a 10 kg ball fired from a cannon at 100 m/sec. What mass did Franklin assign to the light particle?

9.10 In a baseball game, both the bunt and the long outfield fly are sacrifice hits. Contrast the collision processes which make them.

9.11 In the light of your knowledge of the relationship between momentum and force, comment on reports about unidentified flying objects (UFO) turning sharp corners.

9.12 A hunter fires a gun horizontally at a target fixed to a hillside. Describe the changes of momentum of the hunter, the bullet, the target and the earth. Is momentum conserved in each case?

9.13 A girl on skis (mass of 60 kg including skis) reaches the bottom of a hill going 20 m/sec. What is her momentum? She strikes a snowdrift and stops within 3 seconds. What force does the snow exert on the girl? How far does she penetrate the drift?

9.14 A horizontal conveyor belt is used to transport grain from a bin to a truck. A 50.0-kg bag of grain falls straight from a chute onto the belt every 20 seconds. The velocity of the conveyor belt is 4.0 m/sec.

a) What is the momentum gained or lost by a bag of grain just as it is placed on the belt?
b) What is the average additional force required to drive the belt when carrying grain?

9.15 The text derives the law of conservation of momentum for two bodies from Newton's third and second laws. Is the principle of the conservation of mass essential to this derivation? If so, where does it enter?

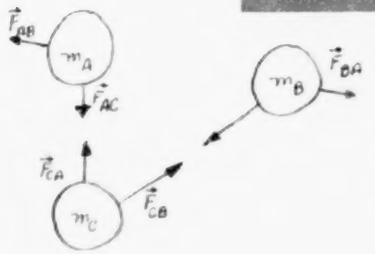
9.16 If mass remains constant, then $\Delta(mv) = m(\Delta v)$. Check this relation for the case where $m = 3$ units, $v' = 6$ units and $v = 4$ units.

9.17 Equation (9.1), $m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}\,'_A + m_B\vec{v}\,'_B$, is a general equation. For a loaded cannon where the subscripts c and s refer to cannon and shell respectively,

a) what are the values of v_c and v_s before firing?
b) what is the value of the left hand side of the equation before firing? after firing?
c) compare the magnitudes of the momenta of cannon and shell after firing.
d) compare the ratios of the speeds and the masses of shell and cannon after firing.
e) a 10 kg shell has a speed of 1000 m/sec. What is the recoil speed of 1000 kg cannon?

9.18 Newton's laws of motion lead to the law of conservation of momentum not only for two-body systems, as was shown in Sec. 9.4, but for systems consisting of any number of bodies. In this problem you are asked to repeat the derivation of Sec. 9.4 for a three-body system.

The figure shows three bodies, with masses m_A, m_B and m_C, exerting forces on one another. The force exerted on body A by body B is $\vec{F}_{AB}$; the force exerted on body C by body A is $\vec{F}_{CA}$, etc.



Using the symbol $\vec{p}$ to represent momentum, we can write Newton's second law as:

$$\vec{F}_{net}\,\Delta t = \Delta(m\vec{v}) = \Delta\vec{p}.$$

Applied to body A, the second law says:

$$(\vec{F}_{AB} + \vec{F}_{AC})\,\Delta t = \Delta\vec{p}_A.$$

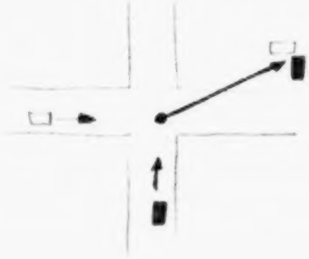
a) Copy the last equation above and write corresponding equations for body B and body C.

b) According to Newton's third law, $\vec{F}_{AB} = -\vec{F}_{BA}$, $\vec{F}_{AC} = -\vec{F}_{CA}$ and $\vec{F}_{BC} = -\vec{F}_{CB}$. Combine these relations with the three equations of (a) to obtain $\Delta\vec{p}_A + \Delta\vec{p}_B + \Delta\vec{p}_C = 0$.

c) Show that the result of (b) is equivalent to $\vec{p}_A + \vec{p}_B + \vec{p}_C = \vec{p}\,'_A + \vec{p}\,'_B + \vec{p}\,'_C$.

The last equation says that the momentum of the three-body system is constant during the time interval Δt.

9.19 A police report of an accident describes two vehicles colliding (inelastically) at a right-angle intersection. The cars skid to a stop as shown. Suppose the masses of the cars are approximately the same. Which car was traveling faster at collision? What information would you need in order to calculate the speed of the automobiles?

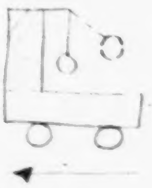


9.20 Two pucks on a frictionless horizontal surface are joined by a spring. Can they be considered an isolated system? How do gravitational forces exerted by the earth affect your answer? What about forces exerted on the planet earth? Do you wish to reconsider your answer above? How big would the system have to be in order to be considered completely isolated?

9.21 Two balls, one of which has three times the mass of the other, collide head-on, each moving with the same speed. The more massive ball stops, the other rebounds with twice its original speed. Show that both momentum and vis viva are conserved.

9.22 If both momentum and vis viva are conserved and a ball strikes another three times its mass at rest, what is the velocity of each ball after impact?

9.23 A student in a physics class, having learned about elastic collisions and conservation laws, decides that he can make a self-propelled car. He proposes to fix a pendulum on a cart, using a "super-ball" as a pendulum bob. He fixes a block to the cart so that when the ball reaches the bottom of the arc, it strikes the block and rebounds elastically. It gives the cart a series of bumps that propel it along.



a) Will his scheme work? (Assume that the "super-ball" is perfectly elastic.) Give reasons for your answer.

b) What would happen if the cart had an initial velocity in the forward direction?

c) What would happen if the cart had an initial velocity in the backward direction?

9.24 A billiard ball moving 0.8 m/sec collides with the cushion along the side of the table. The collision is head-on and perfectly elastic. What is the momentum of the ball before impact? After impact? What is the change in momentum? Is momentum conserved? (Pool sharks will say that it depends upon the "English" [spin] that the ball has, but the problem is much simpler if you neglect this condition.)

9.25 Fill in the blanks:

OBJECT	MASS (kg)	VELOCITY (m/sec)	MOMENTUM kg-m/sec	VIS VIVA kg-m^2/sec^2
baseball	0.14	30.0		
hockey puck		50.0	8.55	
superball	0.050	1.5		
Corvette	1460			1.79×10^6
mosquito			2.0×10^{-5}	4.0×10^{-6}

9.26 You have been given a precise technical definition for the word momentum. Look it up in a big dictionary and record its various uses. Can you find anything similar to our definition in these more general meanings? Why was the word momentum selected as the name for quantity of motion?

9.27 Descartes defined the quantity of motion of an object as the product of its mass and its speed. Is his quantity of motion conserved as he believed it was? If not, how would you modify his definition so it would be conserved?

Chapter 10 Energy

Section		Page
10.1	Work and kinetic energy	37
10.2	Potential energy	40
10.3	Conservation of mechanical energy	42
10.4	Forces that do no work	44
10.5	Heat energy and the steam engine	46
10.6	James Watt and the Industrial Revolution	51
10.7	The experiments of Joule	58
10.8	Energy in biological systems	60
10.9	The law of conservation of energy	64

10.1 *Work and kinetic energy*. In everyday language we say that we are "playing" when we are pitching, catching and running on the softball field; when we are sitting at a desk solving physics problems, we are "working." A physicist would disagree. He would say that while studying we are doing very little work, whereas on the softball field we are doing a great deal of work. "Doing work" means something very definite to a physicist: it means "exerting a force on an object while the object moves in the direction of the force." Thus when you throw a softball, you exert a large force on it while it moves forward about a yard; you do a large amount of work. By contrast, turning pages of a textbook requires you to exert only a small force, and the page doesn't move very far; you don't do much work.

Suppose you are employed in a factory to lift boxes from the floor to a conveyor belt at waist height. Both you and a physicist would agree that you are doing work. It seems like common sense to say that if you lift two boxes at once, you do twice as much work as you do if you lift one box. It also seems reasonable to say that if the conveyor belt were twice as high above the floor, you would do twice as much work to lift a box to it. The work you do depends on both the magnitude of the *force* you must exert on the box and the *distance* through which the box moves.

Note that the work you do does not depend on how *fast* you do your job.

The physicist's definition of work is in agreement with these common-sense notions. The work done on an object is defined as the product of the magnitude, F, of the force exerted on the object and the distance d that the object moves while the force is being exerted:

$$W = Fd. \qquad (10.1)$$

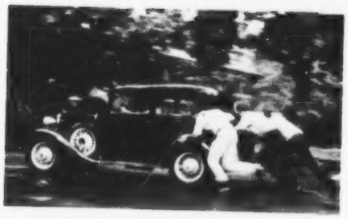
A more general definition of work is given in Sec. 10.4.

So far we have not indicated that the concept of work has any use. You probably realize by now, however, that in physics the only concepts that are defined are those which are useful. Work is indeed a useful concept; it is, in fact, crucial to an understanding of the concept of energy.

There are a great many forms of energy. A few of them will be discussed in this chapter. We will define them, in the sense that we will tell how they can be measured and how they can be expressed algebraically. The *general* concept of energy is very difficult to define; in this course we shall not attempt to do so. On the other hand, to define some *particular* forms of energy is straightforward, and it is through the concept of work that the definitions can be made.

The significance of the concept of work is that work represents an amount of energy transformed from one form to another. For example, when you throw a softball (do work on it), you transform *chemical* energy, which your body obtains from food, into *energy of motion*. When you lift a stone (do work on it), you transform chemical energy into *gravitational potential energy*. If the stone is released, the earth pulls it downward (does work on it), and gravitational potential energy is transformed into energy of motion. When the stone strikes the ground, it pushes the ground downward (does work on it), and energy of motion is transformed into *heat*.

On the following page it is shown how we can use the definition of work (Eq. 10.1), together with Newton's laws of motion, to get an expression for what we have called "energy of motion." It turns out that if F is the magnitude of the *net* force exerted on an object of mass m while the object moves a distance d in the direction of the force, and if the object is initially at rest, then



$$Fd = \frac{1}{2} mv^2 \tag{10.2}$$

where v is the speed of the object after it has moved the distance d.

We recognize the left side of Eq. (10.2) as the work done on the object by whatever exerted the force. (On the following page it is "you" who do the work on the object.) The work done on the object equals the amount of energy transformed from some form (chemical energy, for example, as on the facing page into energy of motion of the object. So the right side of Eq. (10.2) must be the expression for the energy of motion of the object. Energy of motion is called *kinetic energy*.

The Greek word "kinetos" means "moving."

The kinetic energy of an object, therefore, is defined as one half the product of its mass and the square of its speed.

$$(KE) = \frac{1}{2} mv^2. \tag{10.3}$$

Equation (10.2) says that the work done on the object equals its final kinetic energy. This is the case if the object was initially at rest, that is, if its initial kinetic energy was zero. More generally the object may already be moving when the net force is applied. In that case the work done on the object equals the *increase* in its kinetic energy.

$$Fd = \Delta(KE), \tag{10.4}$$

where $\Delta(KE) = (\frac{1}{2} mv^2)_{\text{final}} - (\frac{1}{2} mv^2)_{\text{initial}}$.

The name of the unit of energy and work commemorates J.P. Joule, a nineteenth century English physicist, famous for his experiments showing that heat is a form of energy (see Sec. 10.7). The word is pronounced like "jewel."

Since work is defined as the product of a force and a distance, its units in the mks system are newtons-times-meters. A newton-meter is called a *joule*. The unit of energy is thus one joule (abbreviated 1 J).

Doing Work on a Sled

Suppose a loaded sled of mass m is initially at rest on the horizontal frictionless surface of ice. You, wearing spiked shoes, exert a constant horizontal force $\vec{F}$ on the sled. The weight of the sled is balanced by the upward push exerted by the surface, so $\vec{F}$ is the net force on the sled. You keep pushing (running faster and faster to keep up with the accelerating sled) until the sled has moved a distance d.

Since the net force $\vec{F}$ is constant, the acceleration of the sled is constant. Two equations that apply to motion from rest with constant acceleration are

$$v = at$$

and $$d = \tfrac{1}{2}at^2$$

where a is the acceleration of the body, t is the time interval during which it accelerates (that is, the time interval during which a net force acts on the body), v is the final speed of the body and d is the distance it moves in the time interval t.

According to the first equation

$$t = v/a.$$

If we substitute this expression for t into the second equation, we obtain

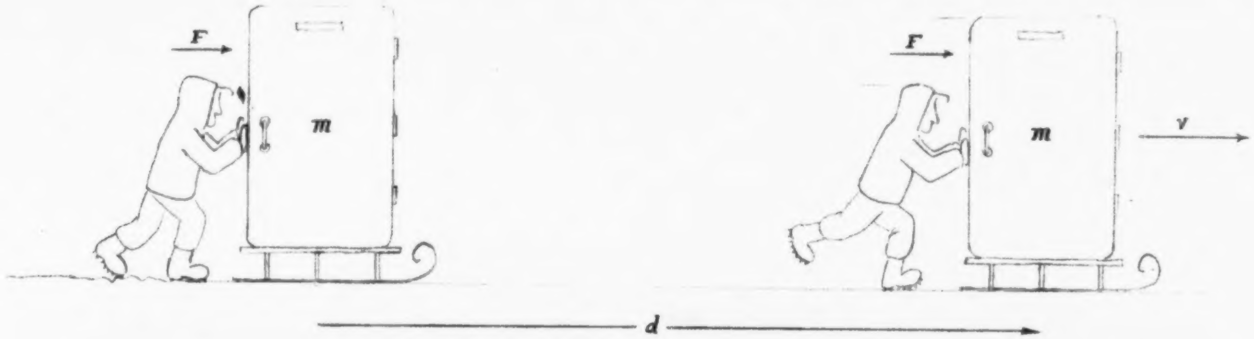
$$d = \tfrac{1}{2}at^2 = \tfrac{1}{2}a\frac{v^2}{a^2} = \tfrac{1}{2}\frac{v^2}{a}.$$

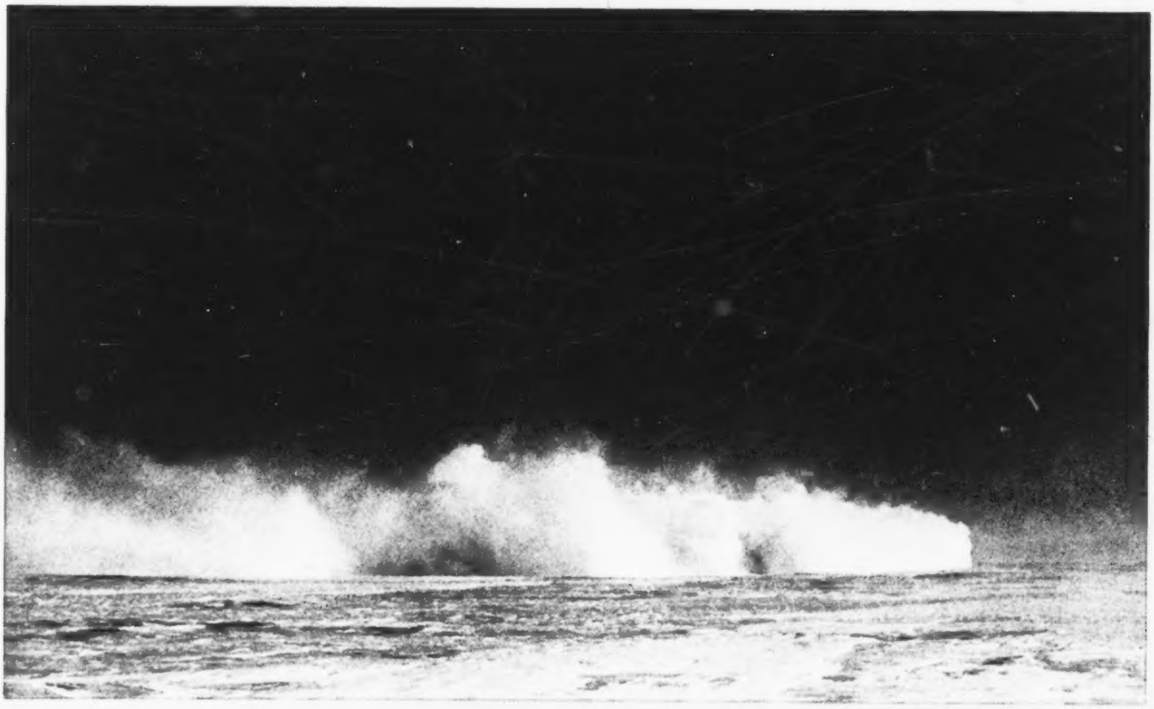
The last equation is a relationship between final speed, (constant) acceleration and distance moved.

The work that you do on the sled is

$$Fd = F(\tfrac{1}{2}\frac{v^2}{a}).$$

But, from Newton's second law, $F = ma$. Therefore, $Fd = (ma) \times (\tfrac{1}{2}\frac{v^2}{a}) = \tfrac{1}{2}mv^2$.





Q1 If a Force $\vec{F}$ is exerted on an object while the object moves a distance d in the direction of the force, the work done on the object is

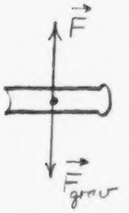
a) Fd.
b) $\frac{1}{2}$ Fd^2.
c) F/d.
d) F.

Q2 The kinetic energy of a body of mass m moving at a speed v is

a) mv.
b) $\frac{1}{2}$mv.
c) 2 mv^2.
d) m^{2}v^2.
e) $\frac{1}{2}$ mv^2.

Q3 If you apply 1 joule to lift a $\frac{1}{2}$ kg book, how high will it rise?

10.2 Potential energy. If work is done on an object, its kinetic energy may increase, as we have seen in the previous section. But it can happen that work is done on an object with no increase in its kinetic energy. For example, to lift a book straight up you must do work on the book, even if you lift it at constant speed so that its kinetic energy stays the same. By doing work you are depleting your body's store of chemical energy. But into what form of energy is it being transformed?

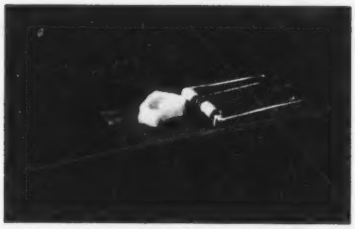


To lift the book at constant speed, you must exert an upward force $\vec{F}$ equal in magnitude to the weight $\vec{F}_{grav}$ of the book.

It is being transformed into gravitational potential energy. Lifting the book higher and higher increases the gravitational potential energy. When the book has been lifted a distance d, the gravitational potential energy has increased by an amount F_{grav} d, where F_{grav} is the weight of the book.

$$\Delta(PE)_{grav} = F_{grav}\, d \qquad (10.5)$$

Potential energy can be thought of as stored energy. If you allow the book to fall, the gravitational potential energy will decrease and the kinetic energy of the book will increase. When the book reaches the floor, all of the stored gravitational potential energy will have been transformed into kinetic energy.



A set mouse-trap illustrates elastic potential energy.

There are other forms of potential energy also. For example, if you stretch a rubber band or a spring, you increase the elastic potential energy. When you release the rubber band, it can deliver the stored energy to a paper wad in the form of kinetic energy.

In an atom the negatively charged electrons are attracted by the positively charged nucleus. If an electron is pulled away from the nucleus, the electrical potential energy of the atom will increase. If the electron is allowed to be pulled back toward the nucleus, the potential energy will decrease and the electron's kinetic energy will increase.

10.3

If two magnets, with north poles facing, are pushed together, the magnetic potential energy will increase. When released, the magnets will move apart, gaining kinetic energy at the expense of potential energy.

The magnetic potential energy does not "belong to" one magnet or the other; it is a property of the system. The same is true of all forms of potential energy. Gravitational potential energy, for example, is a property of the system of the earth and the elevated book.

Q4 If a stone of mass m falls a vertical distance d, the decrease in gravitational potential energy is
- a) md.
- b) ma_g.
- c) $ma_g d$.
- d) $\frac{1}{2} md^2$.
- e) d.

Q5 When you compress a coil spring you do work on it. The elastic potential energy
- a) disappears.
- b) breaks the spring.
- c) increases.
- d) stays the same.
- e) decreases.

Q6 Two positively charged objects repel one another. To increase the electric potential energy, you must
- a) make the objects move faster.
- b) move one object in a circle around the other object.
- c) attach a rubber band to the objects.
- d) pull the objects farther apart.
- e) push the objects closer together.

10.3 Conservation of mechanical energy. In Sec. 10.1 it was stated that the amount of work done on an object equals the amount of energy transformed from one form to another. Perhaps you realized that this statement implies that the amount of energy does not change, that only its form changes.

If a stone falls, for example, there is a continual transformation of gravitational potential energy into kinetic energy. Over any part of its path the decrease in gravitational potential energy is equal to the increase in kinetic energy. On the other hand, if a stone is thrown upwards, then at any point in its path the increase in gravitational potential energy equals the decrease in kinetic energy. For a stone falling or rising, then

Equations (10.6), (10.7), and (10.8) are true only if there is negligible friction.

$$\Delta(PE)_{grav} = -\Delta(KE). \qquad (10.6)$$

Equation (10.6) can be written as

$$\Delta(PE)_{grav} + \Delta(KE) = 0, \qquad (10.7)$$

which says that the change in the total energy ($KE + PE_{grav}$) is zero. In other words, the total energy ($KE + PE_{grav}$) remains constant--it is conserved.

The total energy is conserved also when a guitar string is plucked. As the string is pulled away from its unstretched position, the elastic potential energy increases. When the string is released and allowed to return to its unstretched position, the elastic potential energy decreases while the kinetic energy of the string increases. As the string coasts through its unstretched position and becomes stretched in the other direction, its kinetic energy decreases and the elastic potential energy increases. As it vibrates, then, there is a repeated transformation of elastic potential energy into kinetic energy and back again. Over any part of its motion, the decrease or increase in elastic potential energy is accompanied by an equal increase or decrease in kinetic energy:

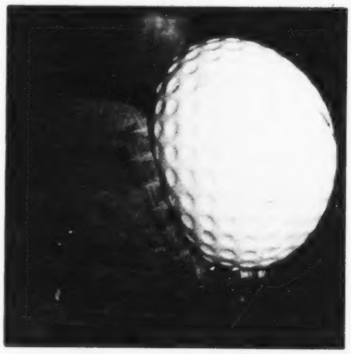
$$\Delta(PE)_{elas} = -\Delta(KE). \tag{10.8}$$

Again, at least for short times, the total energy (KE + PE_{elas}) remains constant—it is conserved.

Whenever potential energy is transformed into kinetic energy, or vice versa, or whenever potential energy is transformed into another form of potential energy, the total energy (KE + PE) does not change. This is the law of conservation of mechanical energy.

The law of conservation of mechanical energy is a consequence of the definition of kinetic energy and the definition of change of potential energy, together with Newton's laws of motion. In a sense, the law of conservation of mechanical energy tells us nothing that we do not already know from Newton's laws of motion. However, there are situations where we simply do not have enough information about the forces involved to apply Newton's laws; it is then that the law of conservation of mechanical energy strongly demonstrates its usefulness.

An elastic collision is a good example of a situation where we often cannot apply Newton's laws of motion, because we do not know the force that one object exerts on the other during the collision. But we do know that during the actual collision, kinetic energy of the colliding objects is transformed into elastic potential energy as the objects distort one another. Then all the elastic potential energy is transformed back into kinetic energy, so that when the objects have separated, their total kinetic energy is the same as it was before the collision.



During its contact with a golf club, a golf ball is distorted, as is shown in the high-speed photograph. As the ball recovers its normal spherical shape, elastic potential energy will be transformed into kinetic energy.

It is important to point out that application of Newton's laws, if this were possible, would provide more detailed information; namely, the speed of each of the objects. The law of conservation of mechanical energy gives us only the total kinetic energy of the objects after the collision, not the kinetic energy of each object separately.

Mechanical energy is the concept that Leibniz had been

groping for in his thinking about the conservation of "force". His *vis viva* differs only by a factor of one half from the modern concept of kinetic energy. He had an idea of gravitational potential energy also, in that he maintained that an object elevated above the ground still possesses "force". He even measured its "force" by the product of its weight and its height above the ground, just as we measure gravitational potential energy.

Q7 As a stone falls
- a) its kinetic energy is conserved.
- b) gravitational potential energy is conserved.
- c) kinetic energy changes into gravitational potential energy.
- d) no work is done on the stone.
- e) there is no change in the total energy.

Q8 In what part of its motion is the kinetic energy of a vibrating guitar string the greatest? When is the elastic potential energy the greatest?

Q9 If a guitarist gives the same amount of elastic potential energy to a bass string and to a treble string, which one will gain more kinetic energy when released? (The mass of a meter of bass string is greater than that of a meter of treble string.)

$W = Fd$ (Eq. 10.1)

10.4 Forces that do no work. In Sec. 10.1 we defined the work done on an object as the product of the magnitude of the force exerted on the object and the distance through which the object moves while the force is being applied. This definition is satisfactory so long as the object moves in the same direction as the force, as in all the examples we have done so far.

But there are cases where the direction of motion and the direction of the force are not the same. For example, suppose you carry a book horizontally (so that its potential energy does not change), with constant speed (so that its kinetic energy does not change). Since there is no change in the energy of the book, it must be that you did no work on the book. Yet you did exert a force on the book and the book did move through a distance.

The force and the distance, however, were at right angles. You exerted a vertical force on the book—upwards to balance its weight, and the book moved horizontally. We can conclude that if a force is exerted on an object while the object moves at right angles to the direction of the force, the force does no work. In order to include such cases, we must define work more precisely than we did in Sec. 10.1.

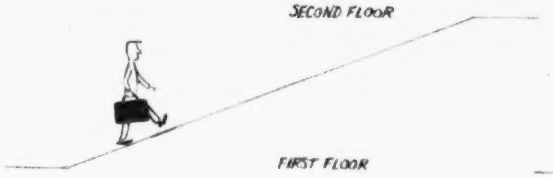
If you do not understand what is meant by a "component of force," work through the section on components in the programmed instruction booklet on vectors.

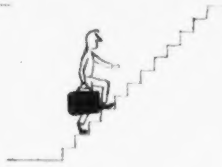
The work done on an object is defined as the product of the component of the force on the object in the direction of motion of the object and the distance through which the object moves while the force is being applied:

$$W \equiv F_{\parallel} d, \qquad (10.9)$$

where $F_{\parallel}$ stands for the component of force parallel to the direction of motion.

According to Eq. (10.9) no work is done on a suitcase, for example, when it is moved at constant speed along a horizontal path. Work is done on it when it is lifted higher above the ground, increasing its gravitational potential energy. Whether it is lifted at constant speed straight up to the second floor in an elevator, or whether it is carried at constant speed up a flight of stairs or a ramp, there is the same increase in gravitational potential energy and hence the same amount of work is done on the suitcase.





Similarly, whether the suitcase falls out of the second floor window, or tumbles down the stairs, or slides down the ramp, the decrease in the gravitational potential energy is the same; the earth does the same amount of work on it: $W = ma_g d$. If there are no frictional forces, the kinetic energy of the suitcase will be the same when it reaches the first floor, no matter how it got there.

The change in gravitational potential energy depends only on the change in the vertical position of the suitcase, and not on the path along which it moves. The same is true of other forms of potential energy: change in potential energy depends on the initial and final positions, and not on the path.

If there are frictional forces on the suitcase or other objects in the system, work has to be done against these by the objects as they move. This work warms up the stairs, the air, the motor, or the body of the man; the change in gravitational potential energy, however, is the same whether there is friction or not.

Q10 When Tarzan swings from one tree to another on a hanging vine, does the vine do work on him?

Q11 No work is done when
- a) a heavy box is pushed at constant speed along a rough horizontal floor.
- b) a nail is hammered into a board.
- c) there is no component of force parallel to the direction of motion.
- d) there is no component of force perpendicular to the direction of motion.

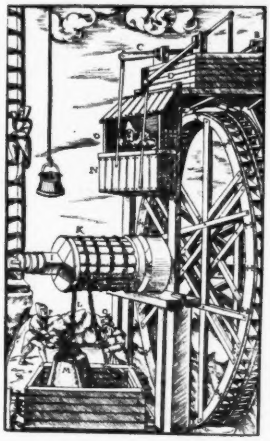
Q12 A suitcase is carried by a porter up a ramp from the ground to the second floor. Another identical suitcase is lifted from the ground to the second floor by an elevator. Compare the increase in gravitational potential energy in the two cases.

10.5 Heat energy and the steam engine. Suppose that a book lying on the table is given a shove and slides along the horizontal surface of the table. If the surface is rough, so that it exerts a frictional force on the book, the book will not keep moving for very long; its kinetic energy gradually disappears. But there is no corresponding increase in potential energy! It appears that in this example mechanical energy is not conserved.

Close examination of the book and the surface of the table, however, would show that they are warmer than before. The disappearance of kinetic energy is accompanied by the appearance of heat. This suggests—but by no means proves—that the kinetic energy of the book was transformed into heat; that is, that heat is one form of energy.

Although scientists today believe that heat is indeed a form of energy, it was not until the middle of the nineteenth century that the view of heat as a form of energy became widely accepted. In Sec. 10.9 we will discuss the reasons for its acceptance at that time, and we will see that one of the reasons was the increased knowledge of heat and work that was gained in the development—for very practical reasons—of the steam engine.

Until about 200 years ago, most work was done by people or animals. Wind and water were exploited also, but generally both were unreliable sources of energy and could not easily be used at the times and places where they were needed. In the eighteenth century there was a great need for an economical method of pumping water out of mines, which otherwise became flooded and had to be abandoned. The steam engine was developed to meet this very practical need.



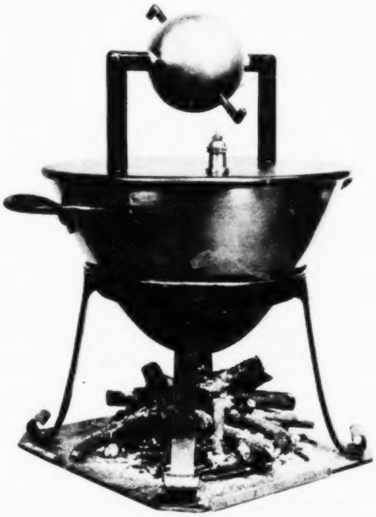
The steam engine is a device for converting the energy of some kind of fuel (the chemical energy of coal or oil, for example, or the nuclear energy of uranium) into heat energy, and then converting the heat energy into mechanical energy. This mechanical energy can then be used directly to do work, or (as is more common now) can be transformed into electrical energy. In typical twentieth-century industrial societies, most of the energy used in factories and homes is electrical energy. Although waterfalls are used in some part of the country, it is steam engines that generate most of the electrical energy used in the United States today.

The generation and transmission of electrical energy, and its conversion into mechanical energy, will be discussed in Chapter 15. Here we are going to turn our attention to the central link in the chain of energy-conversions, the steam engine. As we will see, the development of the steam engine did not take place because of an application of physics to engineering; on the contrary, the engineering analysis of the steam engine led to new discoveries in physics.

Since ancient times it had been known that heat could be used to produce steam, which could then do mechanical work. The "aeolipile," invented by Heron of Alexandria about 100 A.D., worked on the principle of Newton's third law (this principle, of course, had not been announced as a law of physics in the form we now know it). The rotating lawn sprinkler works the same way except that the driving force is due to water pressure rather than steam pressure.

Heron was a Greek inventor and mathematician who lived in Alexandria while it was occupied by the Romans. His books survived through the middle Ages and were circulated widely in Europe during the sixteenth century.

A rotating lawn sprinkler.

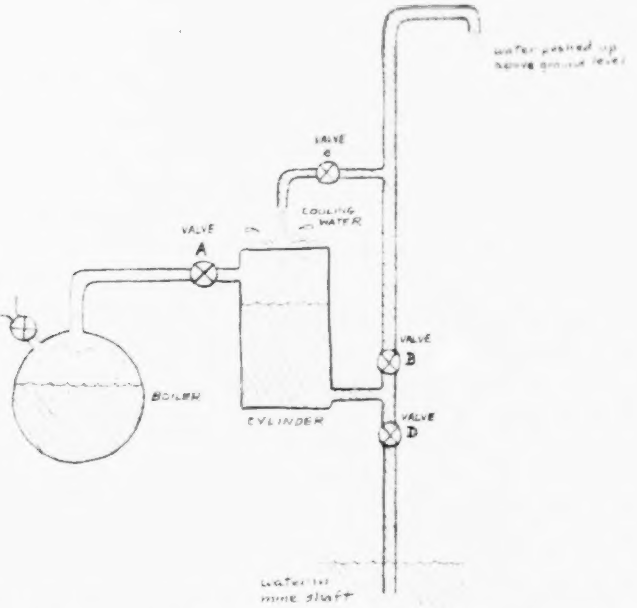


A model of Heron's aeolipile. Steam produced in the boiler escapes through the nozzles on the sphere, causing it to rotate.

Heron's aeolipile was a toy, meant to entertain rather than do any useful work. Perhaps the most "useful" application of steam to do work in the ancient world was a device invented by Heron to "magically" open a door in a temple when a fire was built on the altar. Not until late in the eighteenth century, however, were commercially successful steam engines invented.

Today we would say that a steam engine uses up its supply of heat energy to do work; that is, it converts heat energy into mechanical energy. But many inventors in the eighteenth and nineteenth centuries did not think of heat in this way. They regarded heat as a substance which could be used over and over again to do work, without being used up itself. The fact that these early inventors held ideas about heat which we do not now accept did not prevent them from inventing engines that actually worked. They did not have to learn the correct laws of physics before they could be successful engineers. In fact the sequence of events was just the opposite: steam engines were developed first by practical men who cared more about making money than they did about science. Later on, men who had both a practical knowledge of what would work as well as a curiosity about how it worked, made new discoveries in physics.

10.5



The first commercially successful steam engine was invented by Thomas Savery (1650-1715), an English military engineer, to pump water from mine shafts.

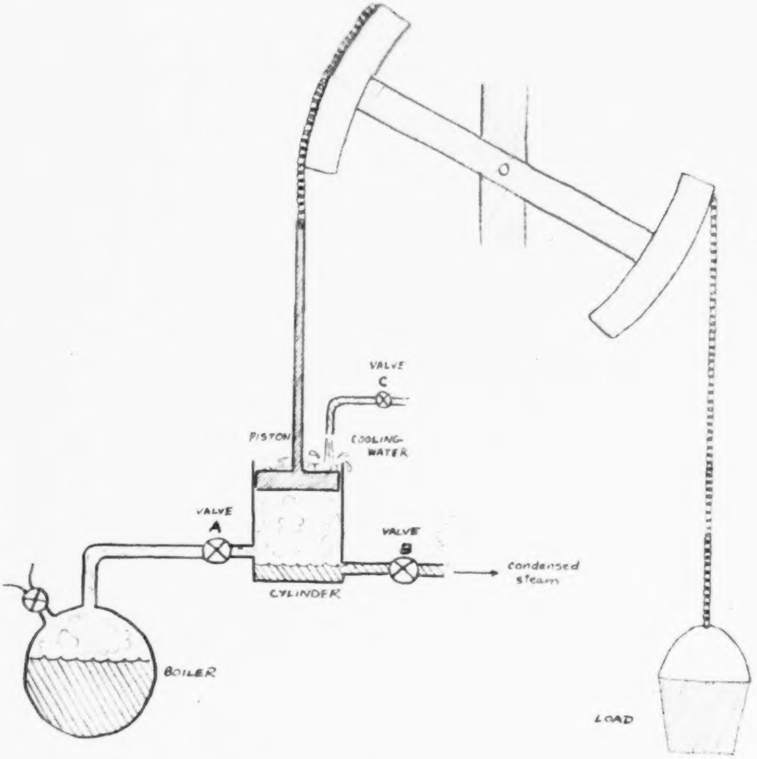
In the Savery engine the water in the mine shaft is connected by a pipe and valve D to a chamber called the cylinder. With valve D closed and valve B open, high-pressure steam from the boiler is admitted to the cylinder through valve A, forcing the water out of the cylinder. Then valve A and valve B are closed and valve D is opened, allowing free access of the water in the mine shaft to the cylinder.

When valve C is opened, cold water pours over the cylinder, cooling the steam in the cylinder and causing it to condense. Since water occupies a much smaller volume than the same mass of steam, a partial vacuum is formed in the cylinder, so that atmospheric pressure forces water from the mine shaft up the pipe and into the cylinder.

The same process, started by closing valve D and opening valves A and B, can be repeated over and over. The engine is in effect a pump, moving water from the mine shaft to the cylinder and, in another step, pushing it from the cylinder to the ground above.

A serious disadvantage of the Savery engine was its use of high-pressure steam, with the attendant risk of boiler or cylinder explosions. This defect was remedied by Thomas Newcomen (1663-1729), another Englishman, who invented an engine which used steam at atmospheric pressure.

10.5



In the original Newcomen engine the load was water being lifted from a mine shaft.

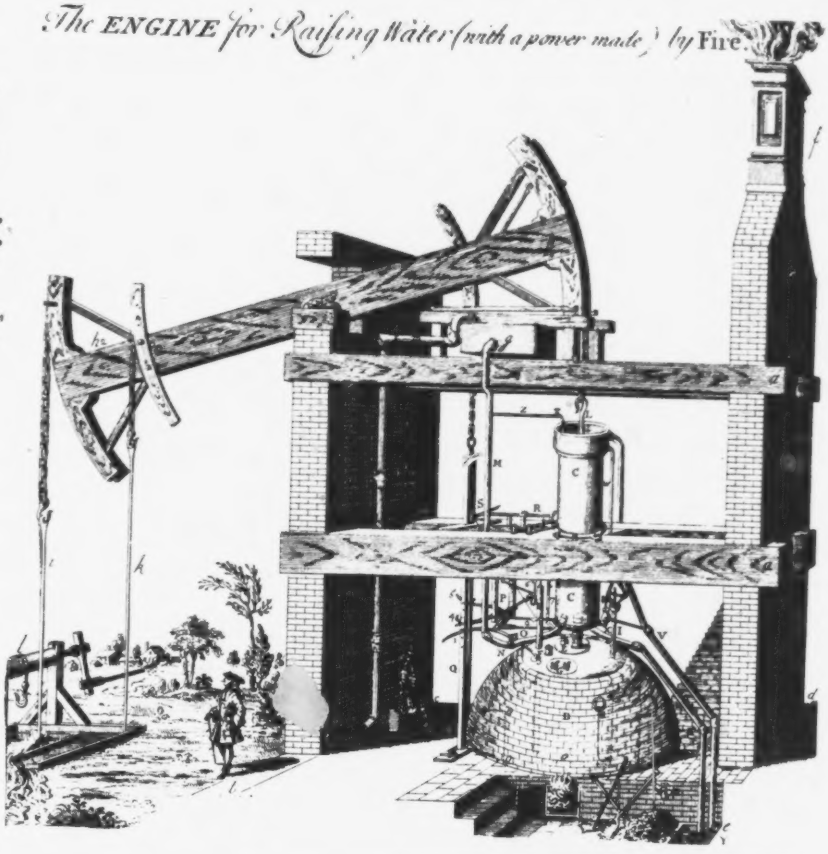
In the Newcomen engine there is a working beam with the load to be lifted on one side and a piston in a cylinder on the other side. The working beam is balanced in such a way that when the cylinder is filled with steam at atmospheric pressure, the weight of the load moves the piston to the upper end of the cylinder. While the piston is coming to this position, valve A is open and valve B is closed.

When the piston has reached its highest position, valve A is closed and valves B and C opened. Cooling water flows over the cylinder and the steam condenses, making a partial vacuum in the cylinder, so that atmospheric pressure pushes the piston down. When the piston reaches the bottom of the cylinder, valves B and C are closed, valve A is opened, and steam reenters the cylinder. The combination of load and steam pressure forces the piston to the top of the cylinder, ready to start the cycle of operations again. Originally it was necessary for someone to open and close the valves at the proper times in the cycle, but later a method was devised for doing this automatically, using the rhythm and some of the energy of the moving parts of the engine itself to control the sequence of operations.

In the words of Erasmus Darwin, the engine
"Bade with cold streams, the
quick expansion stop,
And sunk the immense of
vapour to a drop
Press'd by the ponderous air
the Piston falls
Resistless, sliding through
its iron walls;
Quick moves the balanced
beam, of giant-birth
Wields his large limbs, and
nodding shakes the earth."

10.5

The Newcomen engine was widely used in Britain and other European countries throughout the eighteenth century. By modern standards it was not a very good engine; it burned a large amount of coal but did only a small amount of work at a jerky, slow rate. Nevertheless, because of the demand for machines to pump water from mines, there was a good market even for such an uneconomical steam engine as Newcomen's.


The ENGINE for Raising Water (with a power made) by Fire.

In July 1698 Savery was granted a patent for "A new invention for raiseing of water and occasioning motion to all sorts of mill work by the impellent force of fire, which will be of great use and advantage for drayning mines, serveing townes with water, and for the working of all sorts of mills where they have not the benefitt of water nor constant windes." The patent was good for 35 years and prevented Newcomen from making much money from his superior engine.

Q13 When a book slides to a stop on the horizontal rough surface of a table

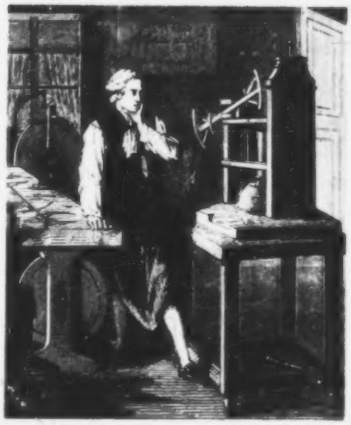
a) the kinetic energy of the book is transformed into potential energy.
b) heat is transformed into mechanical energy.
c) the kinetic energy of the book is transformed into heat energy.
d) the momentum of the book is conserved.

Q14 True or false:

The invention of the steam engine depended strongly on theoretical developments in the physics of heat.

Q15 What economic difficulties did Newcomen encounter in attempting to make his engine a commercial success?

10.6 James Watt and the Industrial Revolution. A greatly improved steam engine originated in the work of a Scotsman, James Watt (1736-1819). Watt's father was a carpenter who had a successful business selling equipment to ships. James was sickly most of his life and gained most of his early education at home. He spent much of his childhood in his father's attic workshop where he developed considerable skill in using tools. He wanted to become an instrument maker and went to London to learn the trade. Upon his return to Scotland in 1757, he obtained a position as instrument maker at the University of Glasgow.

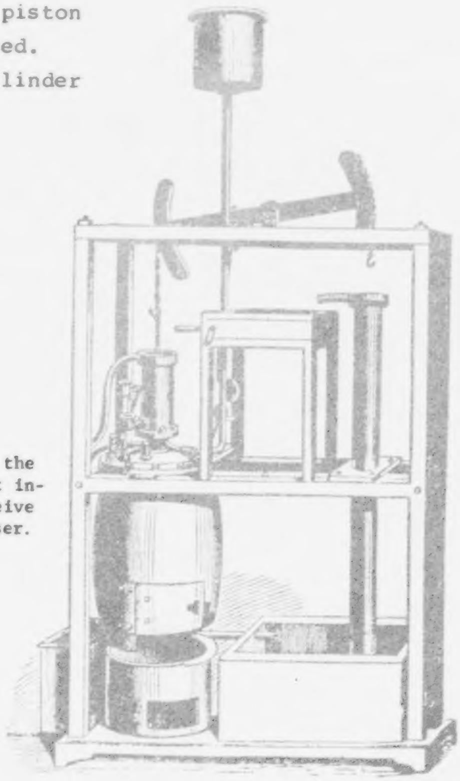


Watt in his workshop contemplating a Newcomen engine. (A romanticized engraving from a nineteenth century volume on technology.)

In the winter of 1763-64 Watt was asked to repair a small model of Newcomen's engine that was used in classes at the university. In familiarizing himself with the model, he observed that Newcomen's engine wasted most of its heat in warming up the walls of its cylinder, which were then cooled down again every time the cold water was injected into the cylinder to condense the steam. This represented a waste of fuel because much of the steam was doing nothing but heating the cylinder walls and then condensing there without doing any work.

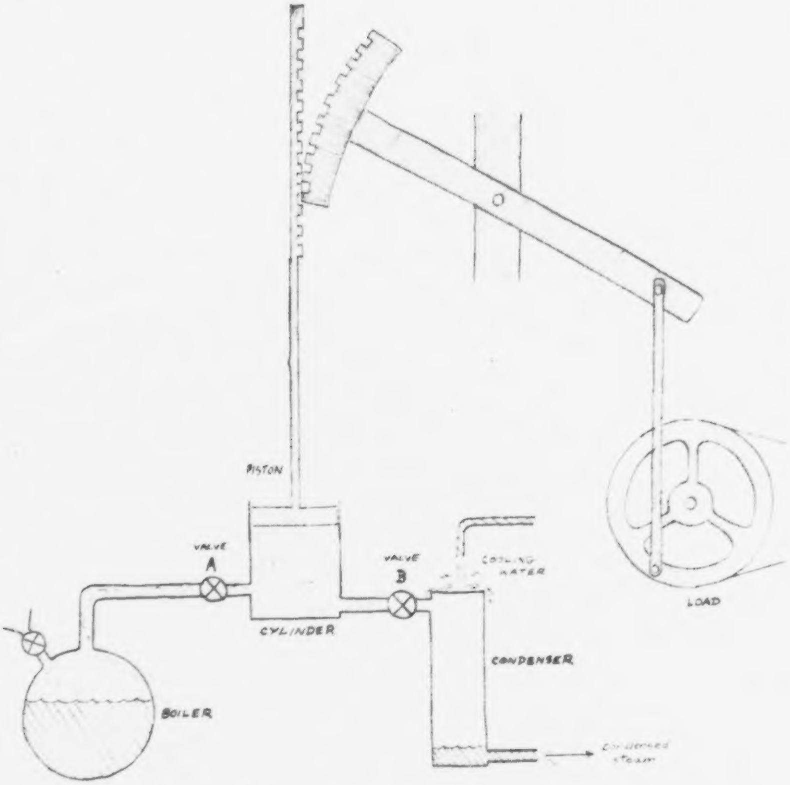
Early in 1765 Watt saw how this defect could be remedied. He devised a new type of steam engine in which the steam in the cylinder, after having done its work of pushing the piston back, was admitted to a separate container to be condensed. The condenser could be kept cool all the time and the cylinder could be kept hot all the time.

James Watt



The actual model of the Newcomen engine that inspired Watt to conceive the separate condenser.

10.6



In the diagram above of Watt's engine, if valve A is open and valve B is closed, steam enters the cylinder at a pressure higher than atmospheric pressure, and pushes the piston upward against the load. When the piston nears the top of the cylinder, valve A is closed to shut off the steam supply. At the same time valve B is opened, so that steam leaves the cylinder and enters the condenser. As the condenser is kept cool by water flowing over it, the steam condenses. This sets up a difference in pressure between the cylinder and the condenser and maintains the flow of steam in the direction toward the condenser. As steam leaves the cylinder the pressure there decreases and the load (plus atmospheric pressure) pushes the piston down. When the piston reaches the bottom of the cylinder, valve B is closed and valve A is opened, admitting steam into the cylinder and starting the next cycle of operations.

Although Watt's invention of the separate condenser might seem to be only a small step in the development of steam engines, it turned out to be a decisive one. The waste of heat was cut down so much by keeping the cylinder always hot that Watt's engine could do more than twice as much work as Newcomen's engine with the same amount of fuel. As a result of this saving in fuel cost, Watt was able to make a fortune by selling or renting his engines to mine owners for the purpose of pumping water.

The fee that Watt charged for the use of his engines depended on their power. Power is defined as the rate of doing work (or the rate at which energy is transformed from one form to another). The mks unit of power is the joule-per-second, which now is appropriately called one watt.

Matthew Boulton (Watt's business partner) proclaimed to Boswell (the famous biographer of Samuel Johnson): "I sell here, Sir, what all the world desires to have: POWER!"

$$1 \text{ watt} \equiv 1 \text{ joule/sec.}$$

James Watt expressed the power of his engines in different units. He found that a strong work horse could lift a 150-pound weight almost four feet in a second; in other words it could do almost 600 foot-pounds of work per second (more precisely 550 foot-pounds per second). Watt defined this as a convenient unit for expressing the power of his engines, the horsepower.

The foot-pound is a unit of work that we shall not use in this course. One foot-pound is defined as the work done when a force of one pound is exerted on an object while the object moves a distance of one foot.

Typical power ratings in horsepower

Man working a pump	0.036
Man turning a crank	0.06
Overshot waterwheel	3
Post windmill	4
Turret windmill	10
Savery steam engine (1702)	1
Newcomen engine (1732)	12
Smeaton's Long Benton engine (1772)	40
Watt engine from Soho (1778)	14
Cornish engine for London water-works (1837)	135
Corliss Philadelphia Exhibition engine(1876)	2500
Electric power station engines (1900)	1000

Source: R. J. Forbes, in C. Singer et al., History of Technology

1 horsepower = 746 watts

Another term which is used to describe the economic value of a steam engine is its duty. The duty of a steam engine is defined as the distance in feet that the engine can lift a load of one million pounds, using one bushel (84 pounds) of coal as fuel.

Duty of Steam Engines

Date	Name	Duty
1718	Newcomen	4.3
1767	Smeaton	7.4
1774	Smeaton	12.5
1775	Watt	24.0
1792	Watt	39.0
1816	Woolf	68.0
1828	Improved Cornish engine	104.0
1834	Improved Cornish engine	149.0
1878	Corliss	150.0
1906	Triple expansion engine	203.0

Source: H. W. Dickinson, Short History of the Steam Engine

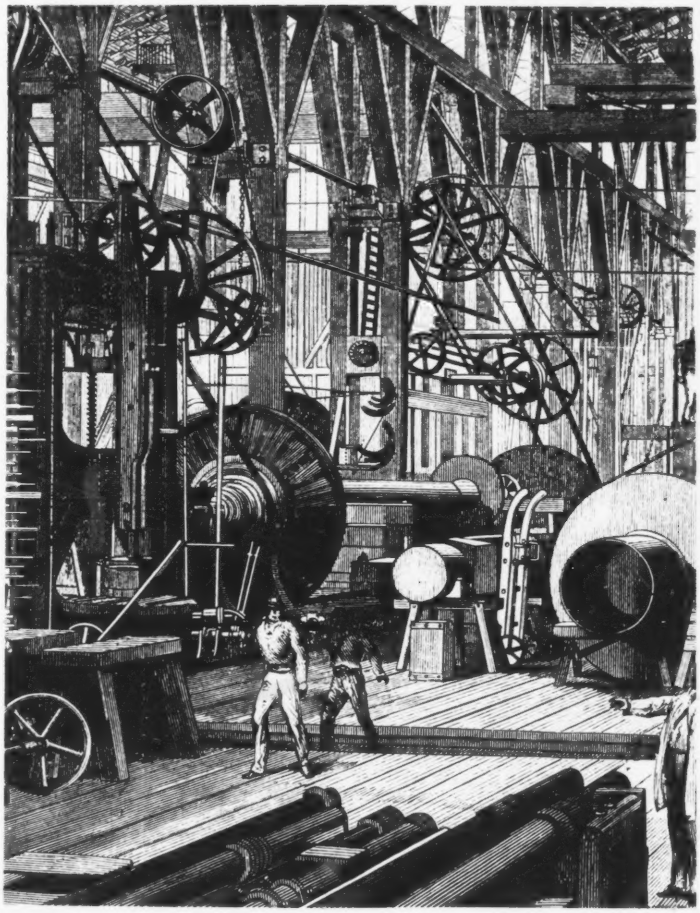
The modern concept of efficiency is related to duty. The efficiency of an engine is defined as the ratio of the work it can do to the amount of energy supplied to it. Efficiency cannot exceed 100%.

10.6

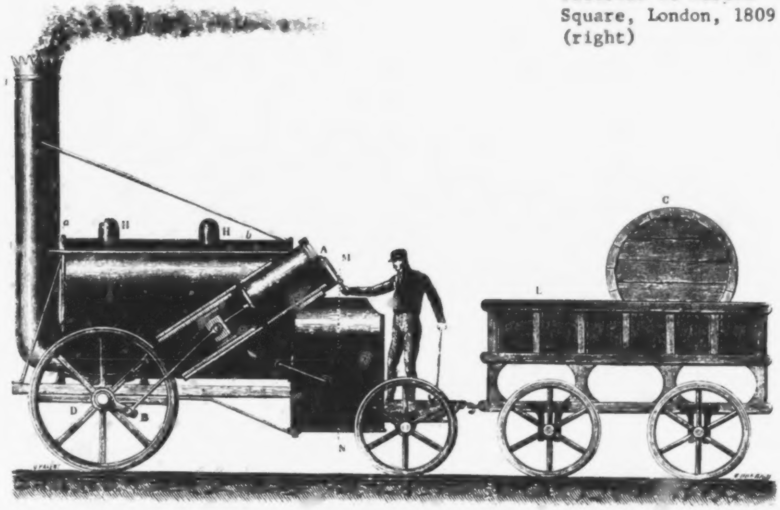
Both power and duty are useful measures of the value of an engine. Its duty tells how much work the engine can do when it uses a given amount of fuel; its power tells how fast the engine can do work.

Watt's invention of the steam engine with separate condenser, so superior to Newcomen's engine, stimulated the development of engines that could do many other kinds of jobs—running machines in factories, driving railway locomotives, steamboats, and so forth. It gave a powerful stimulus to the growth of industry in Europe and America, and thereby helped transform the economic and social structure of Western civilization.

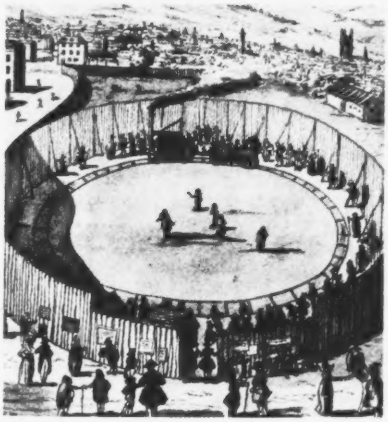
The Industrial Revolution, based on the development of engines and machines for mass production of consumer goods, greatly raised the average standard of living in Western Europe and the United States. Nowadays it is difficult to imagine what life would be like without all the things produced by industry. But not all the effects of industrialization have been beneficial. The nineteenth-century factory system provided an opportunity for some greedy and unscrupulous employers to exploit the workers. These employers made huge profits, while

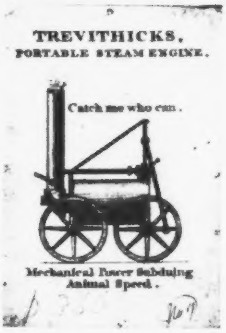


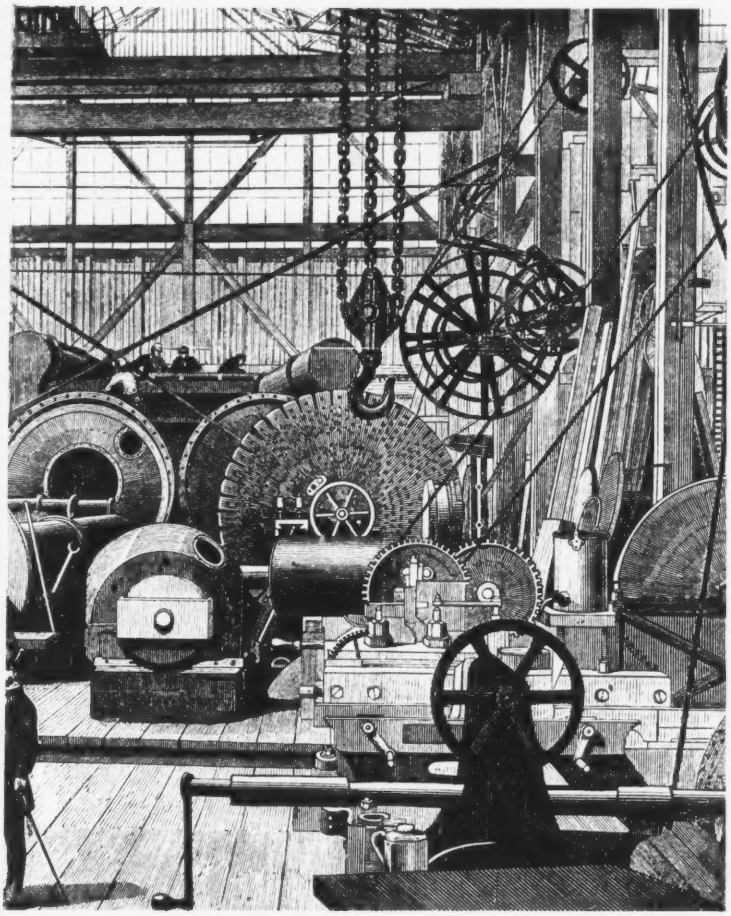
Stephenson's "Rocket" locomotive (below)



Richard Trevithick's railroad at Euston Square, London, 1809 (right)

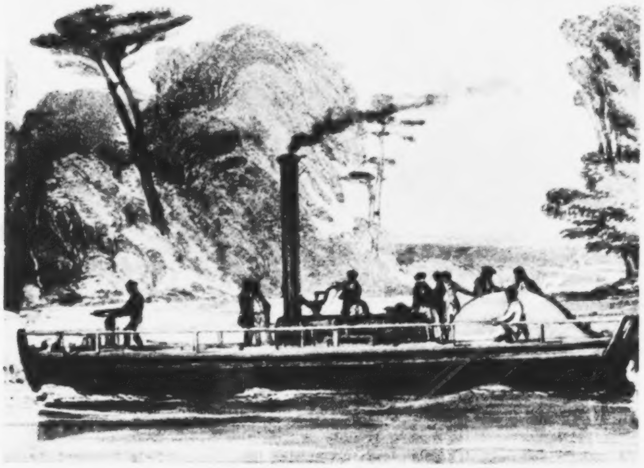



TREVITHICKS,
PORTABLE STEAM ENGINE.
Catch me who can.
Mechanical Power Subduing Animal Speed.



they kept employees and their families on the verge of starvation. This situation, which was especially bad in England early in the nineteenth century, led to demands for reform through government action and legislation. The worst excesses were eventually eliminated.

As more and more people left the farms to work in factories, the conflict between the working class, made up of employees, and the middle class, made up of employers and professional men, became more intense. At the same time, the artists and literary intellectuals began to attack the materialistic tendencies of their society, which was increasingly dominated by commerce and machinery. In some cases they became so fearful of technology that they confused science itself with technical applications and denounced both while refusing to learn anything about them. William Blake asked, sarcastically, "And was Jerusalem builded here/ Among these dark Satanic mills?" John Keats was complaining about science when he asked: "Do not all charms fly/ At the mere touch of cold philosophy?" But not all poets were hostile to science, and even the mystical "nature-philosophy" of the Romantic movement had something to contribute to physics.



The "Charlotte Dundas," the first practical steamboat, built by William Symington, an engineer who had patented his own improved steam engine. It was tried out on the Forth and Clyde Canal in 1801.

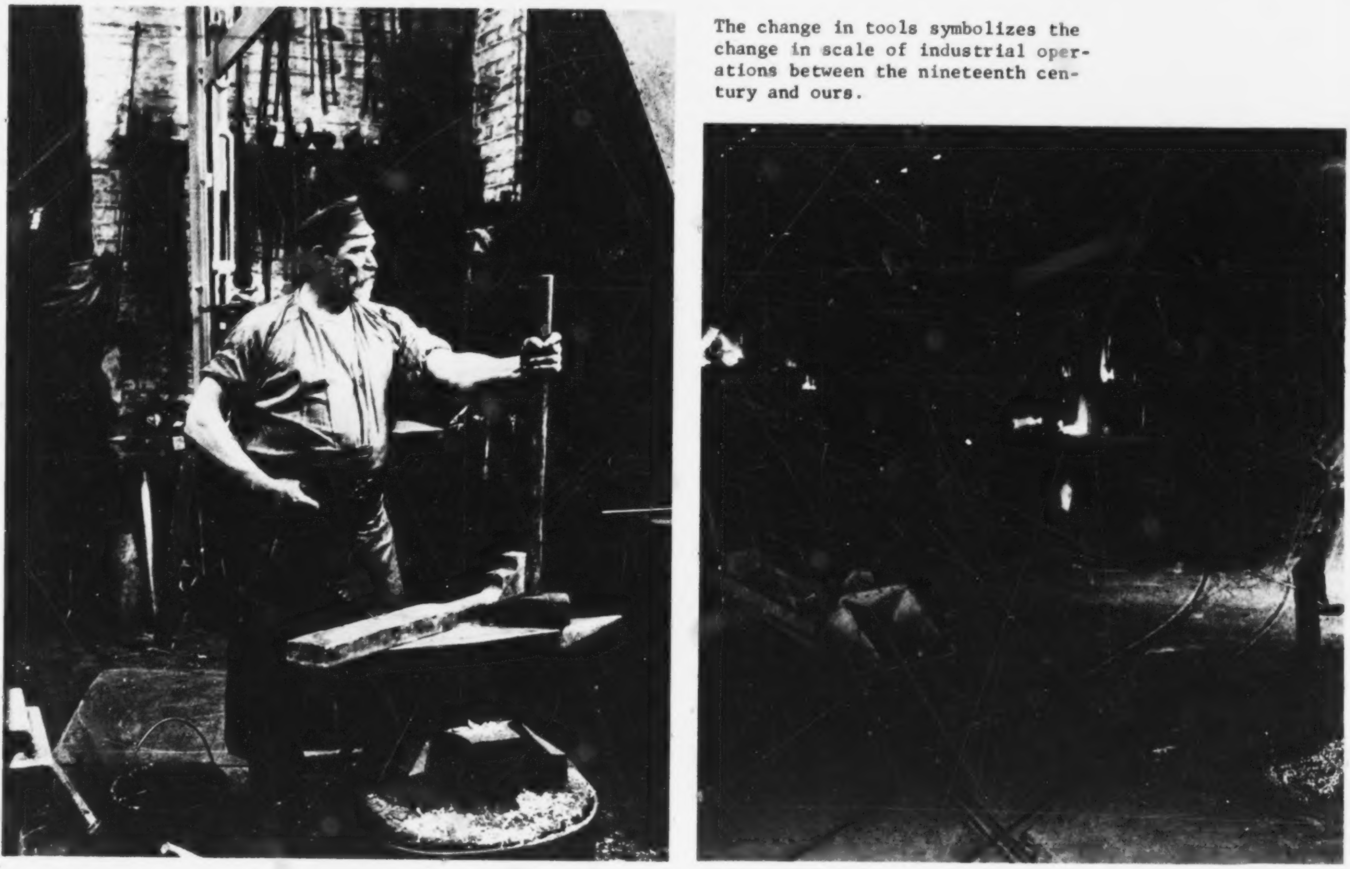
10.6

See "The Steam Engine Comes of Age" in the Project Physics Reader 3.

Although steam engines are no longer widely used as direct sources of power in industry and transportation, they have by no means disappeared. The steam turbine, invented by the English engineer Charles Parsons in 1884, has now largely replaced the older kinds of steam engines, and is now used as the major source of energy in most electric-power stations. The basic principle of the Parsons turbine is simpler than that of the Newcomen and Watt engines: a jet of steam at high pressures strikes the blades of a rotor and makes it go around at high speed.

A description of the type of steam turbine now in operation at power stations in the United States shows the change of scale since Heron's toy:

> The boiler at this station [in Brooklyn, New York] is as tall as a 14-story building. It weighs 3,000 tons, more than a U.S. Navy destroyer. It heats steam to a temperature of 1,050° F and to a pressure of 1,500 pounds per square inch. It generates more than 1,300,000 pounds of steam an hour. This steam runs a turbine to make 150,000 kilowatts of electricity, enough to supply all the homes in a city the size of Houston, Texas. The boiler burns 60 tons (about one carload) of coal an hour.



The change in tools symbolizes the change in scale of industrial operations between the nineteenth century and ours.

The 14-story boiler does not rest on the ground. It hangs—all 3,000 tons of it—from a steel framework. Some boilers are even bigger—as tall as the Statue of Liberty—and will make over 3,000,000 pounds of steam in one hour. This steam spins a turbine that will make 450,000 kilowatts of electricity—all of the residential needs for a city of over 4,000,000 people!

[The Story of Steam (1957), Babcock and Wilcox Co.]

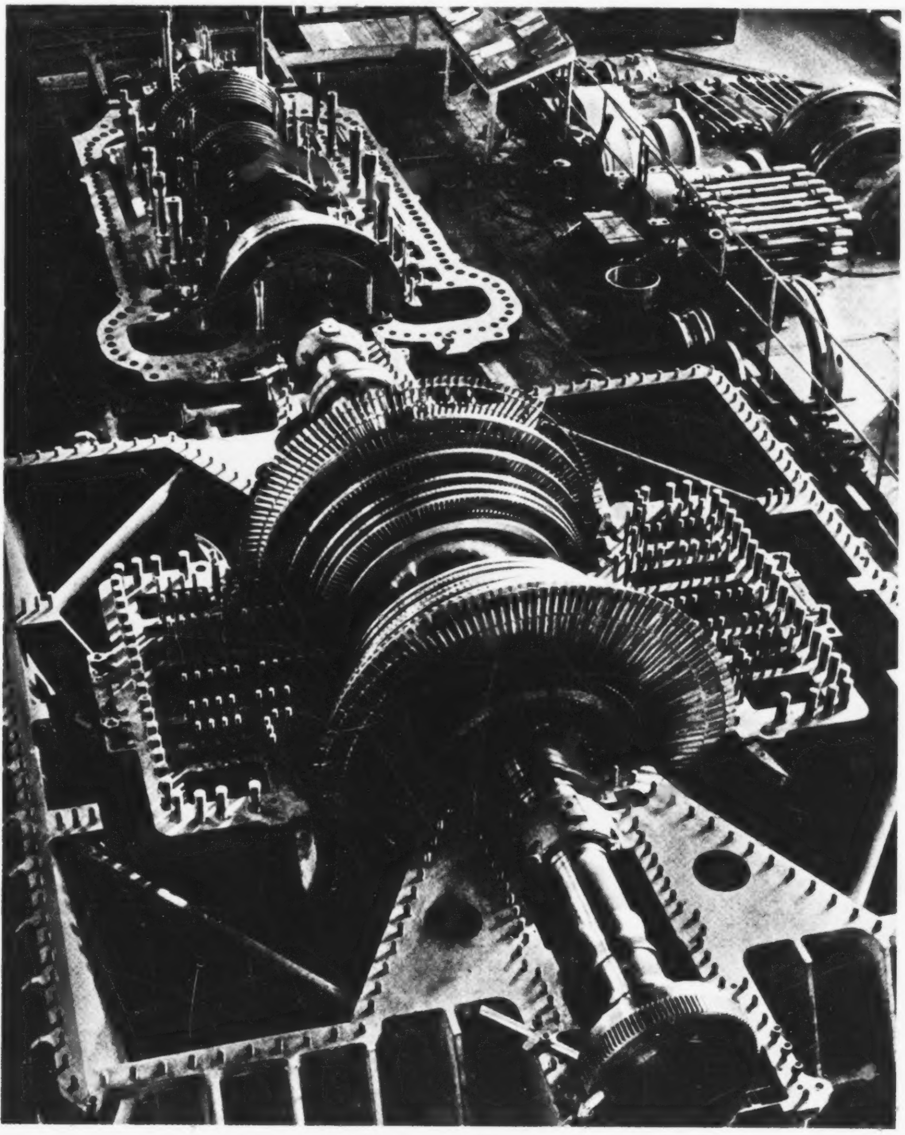
Q16 The purpose of the separate condenser in Watt's steam engine is

a) to save the water so it can be used again.
b) to save the heat so it can be used again.
c) to save fuel by avoiding repeated heating and cooling of the cylinder walls.
d) to keep the steam pressure as low as possible.
e) to make the engine more compact.

Q17 Engine A produces more power than Engine B, but its efficiency is less. This means

a) A is a bigger engine than B.
b) A does more work with the same amount of fuel, but more slowly than B.
c) A does less work with the same amount of fuel, but faster than B.
d) A does more work with the same amount of fuel and faster than B.
e) A does less work with the same amount of fuel and more slowly than B.

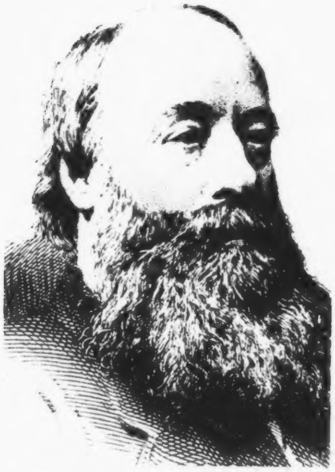
A modern steam turbine for use in a city electric power plant, opened to show the blades of the rotor.



10.7 The experiments of Joule. In the steam engine a certain amount of heat is used to do a certain amount of work. What happens to the heat after it has done the work?

The answer commonly given early in the nineteenth century by most scientists and engineers was that heat can do work if it passes, for example, from steam at high temperature to water at low temperature, but that the amount of heat remains constant. Heat was considered to be a substance, called "caloric," and the total amount of caloric in the universe was thought to be conserved.

According to the caloric theory of heat, the way in which heat can do work is analogous to the way that water can do work if it falls from a high level to a low level, with the total amount of water remaining the same. The explanation was accepted because it seemed plausible, even though no measurements had been made of the amount of heat before and after it did work.



James Prescott Joule

Joule was the son of a wealthy Manchester brewer. He did his research at home and it is said to have been motivated by the desire to develop more efficient engines for the family brewery.

There were some dissenting voices; some people favored the view that heat was a form of energy. One of the scientists who held this view was the English physicist James Prescott Joule (1818-1899).

During the 1840's Joule conducted a long series of experiments designed to show that heat is a form of energy. Joule reasoned that if it could be demonstrated in a variety of different experiments that the same decrease in mechanical energy always resulted in the appearance of the same amount of heat, that would mean that heat is a form of energy.

Electrical technology was still a new field in Joule's day.

For one of his early experiments he constructed a simple electric generator, which was driven by a falling weight. The electric current that was generated heated a wire immersed in water. From the distance the weight descended, he could calculate the decrease in gravitational potential energy, while the mass of the water and its temperature rise gave him the corresponding amount of heat produced. In another experiment he compressed gas in a bottle immersed in water, measuring the amount of work done to compress the gas and measuring the amount of heat delivered to the water.

His most famous experiments were performed with an apparatus in which descending weights caused a paddle-wheel to turn in a container of water. He repeated this experiment many times, constantly improving the apparatus and refining his analysis of the data. In the end he was taking very great care to insulate the container so that heat was not lost to the room; he was measuring the temperature-rise to a fraction of a degree, and in his analysis he was allowing for the small amount of kinetic energy the descending weights had when they reached the floor.

Joule published his results in 1849. He reported

> 1st. That the quantity of heat produced by the friction of bodies, whether solid or liquid, is always proportional to the quantity of [energy] expended. And 2nd. That the quantity of heat capable of increasing the temperature of a pound of water · · · by 1° Fahr. requires for its evolution the expenditure of a mechanical [energy] represented by the fall of 772 lb. through the distance of one foot.

Joule used the word "force" instead of "energy". The scientific vocabulary was still being formed.

The first statement is the evidence that heat is a form of energy. The second statement tells the value of the ratio between the unit of mechanical energy (Joule used the foot-pound) and the unit of heat (Joule used the British Thermal Unit, BTU).

In the mks system the unit of heat is the kilocalorie and the unit of mechanical energy is the joule. Joule's results are equivalent to the statement that 1 kilocalorie equals 4,150 joules. Joule's paddle-wheel experiment, as well as other basically similar ones, has since been performed with great precision. The currently accepted value for the "mechanical equivalent of heat" is

1 kilocalorie = 4184 joules

Units for expressing amount of heat were devised before it was realized that heat is a form of energy. The BTU is the amount of heat that must be added to a pound of water to increase its temperature by 1 Fahrenheit degree. The kilocalorie is the amount of heat that must be added to a kilogram of water to increase its temperature by 1 Celsius (or "Centigrade") degree. In fact, the BTU and the kilocalorie, together with the foot-pound and the joule, are all different units of the same thing: energy.

It is generally accepted now that heat is not a substance, but a form of energy. In particular, it is one of the forms of internal energy, that is, energy associated with the molecules and atoms of which matter is composed. This view of heat will be treated in detail in Chapter 11.

In an inelastic collision, some (or all) of the kinetic energy of the colliding objects is transformed into internal energy, so that after the collision the objects have less kinetic energy than before. (Leibniz had expressed somewhat the same idea when he said that the *vis viva* was "dissipated among the small parts" of the colliding objects.)

Q18 According to the caloric theory of heat, caloric
- a) is a form of water.
- b) can do work when it passes between two objects at the same temperature.
- c) is another name for temperature.
- d) is produced by steam engines.
- e) is a substance that is conserved.

Q19 The kilocalorie is
- a) a unit of temperature.
- b) a unit of energy.
- c) the same as a BTU.
- d) equal to 772 foot-pounds.
- e) an amount of water.

Q20 **In Joule's paddle-wheel experiment, was all the gravitational potential energy used to heat the water?**

10.8 Energy in biological systems. All living things need a supply of energy to maintain life and to carry on their normal activities. Human beings are no exception; we, like all animals, depend on the food we eat to supply us with energy. Human beings are omnivores; they eat both animal and plant materials. Some animals are herbivores, eating only plants, while others are carnivores, eating only animal flesh.

Ultimately, however, all animals, even carnivores, obtain their food energy from plant material. The animal eaten by a lion, for example, has previously dined on plant material (or perhaps on another animal which had eaten plants).

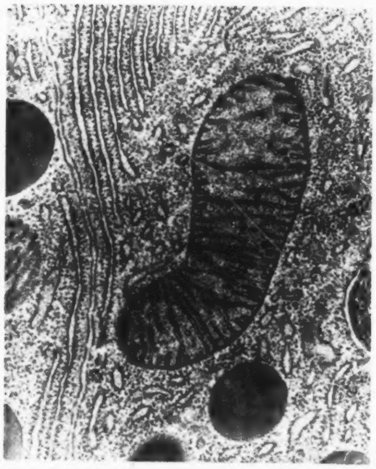
Green plants obtain energy from sunlight. Some of the energy is used to enable the plant to perform the functions of life, but much of it is used to make carbohydrates out of water (H_2O) and carbon dioxide (CO_2). The energy used to synthesize carbohydrates is not lost; it is stored in the carbohydrate molecules in the form of chemical energy.

Carbohydrates are large molecules made of carbon, hydrogen, and oxygen. A simple example is the sugar glucose, the chemical formula for which is $C_6H_{12}O_6$.

The process by which plants synthesize carbohydrates is called photosynthesis. It is still not completely understood; research in the field is continuing. It is known that the synthesis takes place in a large number of small steps, and many of the steps are well understood. When man learns how to photosynthesize carbohydrates without plants, he may be able economically to produce food for the rapidly increasing population of the world. The overall process of producing carbohydrates (the sugar glucose, for example) by photosynthesis can be represented as follows:

carbon dioxide + water + energy $\longrightarrow$ glucose + oxygen

The energy stored in the glucose molecules is used by the animal which eats them to maintain its body temperature, to keep its heart, lungs, and other organs operating, to enable various chemical reactions to occur in the body and to do work on external objects. The process whereby the stored energy is made available is complex. It takes place mostly in tiny bodies called mitochondria which are found in all cells. Each mitochondrion contains catalysts (called enzymes), which, in a series of about ten steps, split the glucose molecules into simpler molecules. In another sequence of reactions these molecules are oxidized, releasing most of the stored energy, and forming carbon dioxide and water.



Electron micrograph of an energy-converting mitochondrion in a bat cell (200,000 times actual size).

The released energy is used to change a molecule called adenosine di-phosphate (ADP) into adenosine tri-phosphate (ATP), a process which requires energy. In short, the chemical energy originally stored in the glucose molecule is eventually stored as chemical energy in ATP molecules.

The ATP molecules pass out of the mitochondrion into the body of the cell (the cytoplasm). Wherever in the cell energy is needed, it can be supplied by an ATP molecule, which releases its stored energy and changes to ADP. Later, in the mitochondrian, the ADP will be converted again to energy-rich ATP.

The overall result of this process is that glucose, in the presence of oxygen, is broken into carbon dioxide and water, with the release of energy:

glucose + oxygen → carbon dioxide + water + energy.

Animals and plants are mutually necessary. The water and carbon dioxide exhaled by animals are used by plants to synthesize carbohydrates, releasing oxygen in the process. The oxygen is used by animals to oxidize the carbohydrates. The energy "used up" by animals (and to a lesser extent, by plants) is continually replenished by energy from sunlight.

Proteins and fats are used to build and replenish tissue, enzymes and padding for delicate organs. They also can be used to provide energy. Both proteins and fats enter into chemical reactions which result in the formation of the same molecules as the splitting carbohydrates; from that point the energy-releasing process is the same as in the case of carbohydrates.

The human body can be regarded as resembling in some respects an engine. Just as a steam engine uses chemical energy stored in coal or oil as fuel, the body uses chemical energy stored in food. In both cases the fuel is oxidized to release its stored energy, violently in the steam engine and gently, in small steps, in the body. In both the steam engine and the body, some of the input energy is used to do work and the rest is used up internally and lost as heat.

Some foods supply more energy than others. The energy stored in food is usually measured in kilocalories, but it could just as well be measured in joules—or even in foot-pounds or British Thermal Units. The table below gives the energy content of some foods.

The chemical energy stored in food can be determined by burning the food in a closed container immersed in water and measuring the temperature rise of the water.

Food	Energy Content
Beef (Hamburger)	3880 kilocalories per kg
Whole Milk	730
Sweet Corn	910
White Rice	1270
Potatoes (boiled, peeled)	890
Wheat (whole meal)	770

Adapted from U.S. Dept. Agric., Agriculture Handbook No. 8, June 1950.

10.8

If the temperature of the body changes by a few degrees, it seriously affects the rate at which important chemical reactions go on in the body.

A large part of the energy you obtain from food is used to keep your body's "machinery" running and to maintain your body's temperature at the correct value. Even when asleep your body uses about one kilocalorie every minute! This amount of energy is needed just to keep alive.

To do work you need a supply of extra energy—and only a fraction of it can be used to do work. The rest is wasted as heat; like any engine, the human body is not 100% efficient. The efficiency of the body when it does work varies with the job and the physical condition and skill of the worker, but probably in no case does it exceed 25%.

In the table below are estimates of the rate at which a healthy college student uses energy in various activities. They were made by measuring the amount of carbon dioxide exhaled, so they show the total amount of food energy use, including the amount necessary just to keep the body functioning.

Activity	Rate of using food energy
Sleeping	1.0 to 1.3 kilocalories per min
Lying down	1.3 to 1.6
Sitting still	1.6 to 1.9
Standing	1.9 to 2.1
Walking	3.8
Running fast	8 to 12
Swimming	9

According to the data in the table, if a healthy college student did nothing but sleep for eight hours a day and lie quietly the rest of the time, he would still need at least 1700 kilocalories of energy each day. There are countries where large numbers of people exist on less than 1700 kilocalories a day. In India, for example, the average is 1630 kilocalories a day.* (In the United States the average is 3100 kilocalories a day.*) About half the population of India is on the very brink of starvation and vast numbers of other people in the world are similarly close to that line.

According to the Statistical Office and Population Division of the United Nations, the total food production of all land and water areas of the earth in 1950 added up to 5760 billion kilocalories a day. The world population in 1950 was about 2.4 billion. If the available food had been equally distributed among all the earth's inhabitants, each would have had 2400 kilocalories a day, only slightly over the minimum required for life.

In 1963 it was estimated that the population of the world would double in the next 35 years, so that by the year 2000, it will be 6 billion. Furthermore the rate at which the population is increasing is itself increasing! The problem of supplying food energy for the world's hungry is rapidly becoming one of the most important problems mankind has ever faced.

---

*U.N. Yearbook of National Accounts Statistics, 1964.

Q21 Animals obtain the energy they need from food, but plants
- a) obtain energy from sunlight.
- b) obtain energy from water and carbon dioxide.
- c) obtain energy from seeds.
- d) do not need any supply of energy.

Q22 The human body has an efficiency of about 20%. This means that
- a) only one-fifth of the food you eat is digested.
- b) four-fifths of the energy you obtain from food is destroyed.
- c) one-fifth of the energy you obtain from food is used to run the "machinery" of the body.
- d) you should spend 80% of each day lying quietly without working.
- e) one-fifth of the energy you obtain from food is used to enable your body to do work on external objects.

"The Repast of the Lion" by Henri Rousseau

The Metropolitan Museum of Art

10.9 The law of conservation of energy. In Sec. 10.3 we introduced the law of conservation of mechanical energy, which can be used only in situations in which no mechanical energy is transformed into heat energy or vice-versa. Some physicists thought there were other forms of energy in addition to kinetic energy and potential energy, but they did not know how to measure them quantitatively.

Early in the nineteenth century developments in science, engineering and philosophy suggested that all forms of energy (including heat) could be transformed into one another and that the total amount of energy in the universe is constant.

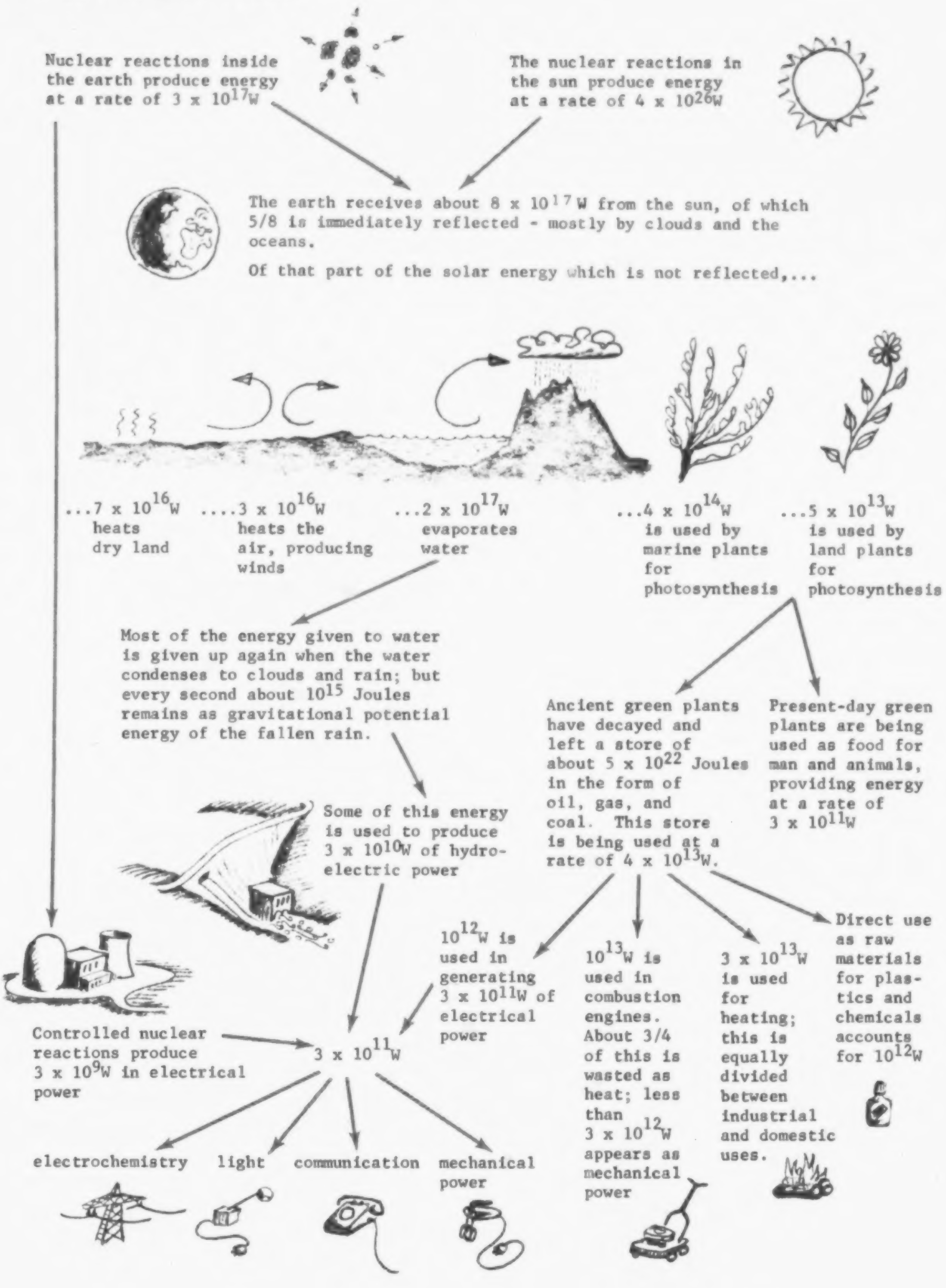
The newly developing science of electricity and magnetism, for example, showed that electricity was related to a number of other phenomena. Volta's invention of the battery in 1800 showed that electricity could be produced in chemical reactions. It was soon found that electric currents could produce heat and light. Oersted discovered in 1820 that an electric current produces magnetic effects. In 1822, Seebeck discovered that if heat is applied to the junction between two metals, an electric current is set up, and in 1831, Faraday discovered electromagnetic induction: a magnet moved near a coil of wire produced an electric current in the wire.

To some speculative minds these discoveries indicated a unity of the phenomena of nature and suggested that they were all the result of the same basic "force". This vague, imprecisely formulated idea bore fruit in the form of the law of conservation of energy: all the phenomena of nature are examples of the transformation from one form to another, without change of quantity, of the same basic thing: energy.

The invention and use of steam engines played a role in the establishment of the law of conservation of energy by showing how to measure changes of energy. Practically from the beginning of their application, steam engines were evaluated by their duty, that is, by how heavy a load they could lift and how high they could lift it when they consumed a certain supply of fuel. In other words, the criterion was how much work an engine could do for the price of a bushel of coal: a very practical consideration which is typical of the engineering tradition in which the steam engine was developed.

The concept of work began to be used in general as a measure of the amount of energy transformed from one form to another (even if the actual words "work" and "energy" were not used) and thus made possible quantitative statements about the transformation of energy. For example, Joule used the work

Energy Conservation on Earth



Nuclear reactions inside the earth produce energy at a rate of 3×10^{17}W

The nuclear reactions in the sun produce energy at a rate of 4×10^{26}W

The earth receives about 8×10^{17} W from the sun, of which 5/8 is immediately reflected - mostly by clouds and the oceans.

Of that part of the solar energy which is not reflected,...

...7×10^{16}W heats dry land

....3×10^{16}W heats the air, producing winds

...2×10^{17}W evaporates water

...4×10^{14}W is used by marine plants for photosynthesis

...5×10^{13}W is used by land plants for photosynthesis

Most of the energy given to water is given up again when the water condenses to clouds and rain; but every second about 10^{15} Joules remains as gravitational potential energy of the fallen rain.

Some of this energy is used to produce 3×10^{10}W of hydro-electric power

Ancient green plants have decayed and left a store of about 5×10^{22} Joules in the form of oil, gas, and coal. This store is being used at a rate of 4×10^{13}W.

Present-day green plants are being used as food for man and animals, providing energy at a rate of 3×10^{11}W

10^{12}W is used in generating 3×10^{11}W of electrical power

10^{13}W is used in combustion engines. About 3/4 of this is wasted as heat; less than 3×10^{12}W appears as mechanical power

3×10^{13}W is used for heating; this is equally divided between industrial and domestic uses.

Direct use as raw materials for plastics and chemicals accounts for 10^{12}W

Controlled nuclear reactions produce 3×10^{9}W in electrical power

3×10^{11}W

electrochemistry

light

communication

mechanical power

done by the descending weights as a measure of the amount of gravitational potential energy transformed into heat energy.

In 1843, on the basis of his early experiments, which were relatively crude compared to his later ones, Joule expressed the conviction that the expenditure of a certain amount of mechanical energy always resulted in the appearance of a definite amount of heat, and that this was an indication of the conservation of what we now call energy. Joule said that he was

> satisfied that the grand agents of nature are by the Creator's fiat indestructible; and that, wherever mechanical force [energy] is expended, an exact equivalent of heat is always obtained.

Joule began his long series of experiments by investigating the duty of electric motors. In this case duty was measured by the work the motor could do when a certain amount of zinc was used up in the battery that ran the motor. Joule's interest was to see whether motors could be made economically competitive with steam engines.

This said, Joule got on with his labors in the laboratory; he was basically a practical man and had little time for theological or metaphysical speculation. Meanwhile, other people were reaching the same conclusion—that the total amount of energy in the universe is constant—on the basis of speculative arguments.

A year before Joule's remark, for example, Julius Robert Mayer, a German physician, had proposed the law of conservation of energy. Unlike Joule, he had done no quantitative experiments, although he had observed physiological processes involving heat and respiration. He used published data on the thermal properties of air to calculate the mechanical equivalent of heat, and obtained about the same value that Joule did.

Julius Robert Mayer

Mayer had been influenced strongly by the German philosophical school now known as Naturphilosophie or "nature-philosophy", which flourished in Germany during the late eighteenth and early nineteenth centuries. Its most influential leaders were Johann Wolfgang von Goethe (1749-1832) and Friedrich Wilhelm Joseph von Schelling (1775-1854). Neither of these men is known today as a scientist. Goethe is frequently regarded as Germany's greatest poet and dramatist, while Schelling is considered only a minor philosopher. Nevertheless, both of them had great influence on the generation of German scientists educated at the beginning of the nineteenth century. The nature-philosophers were closely associated with the Romantic movement in literature, art and music, which was a reaction to what they regarded as the sterility and ethical indifference of the mechanistic world view of Descartes and Newton.

To the nature-philosophers the idea that nature is just a machine made of dead matter in motion was not merely dull, but actually repulsive. They could not believe that the richness of natural phenomena, including Man, with his intellect, his emotions and his aspirations, could be the result of nothing but blind, random motions of particles.

The nature-philosophers believed that the methods of mechanistic science—analyzing nature, mathematizing her into abstraction, torturing her with experiments—gave a totally wrong idea of nature, just as dissecting and measuring a dead flower gives no understanding of the loveliness of a wild rose.

They claimed that they could understand nature as it really is, by direct observation, without complicated and artificial apparatus, just by using their feelings and intuitions. Goethe and Schelling were both very much interested in science and thought that their philosophy could serve as a guide to the discovery of the inner meaning of nature. For Goethe the goal was

> "That I may detect the inmost force which binds the world, and guides its course."

At first glance it would seem that nature-philosophy had little to do with the law of conservation of energy; that law is practical and quantitative, whereas nature-philosophers tended to be speculative and qualitative. In their insistence on searching for the underlying reality of nature, however, the nature-philosophers did influence the discovery of the law of conservation of energy. They believed that the various forces of nature—gravity, electricity, magnetism, etc.—are not really separate from one another, but are simply different manifestations of one basic force. By encouraging scientists to look for connections between different forces (or, in modern terms, between different forms of energy), nature-philosophy stimulated the experiments and theories that led to the law of conservation of energy.

By the time the law was established and generally accepted, however, nature-philosophy was no longer popular. Those scientists who had previously been influenced by it, including Mayer, were now strongly opposed to it. The initial response of some hard-headed scientists to the law of conservation of energy was colored by their distrust of speculative nature-philosophy. For example, William Barton Rogers, founder of the Massachusetts Institute of Technology, wrote home from Europe to his brother in 1858:

> "To me it seems as if many of those who are discussing this question of the conservation of force are plunging into the fog of mysticism."

However, the law of conservation of energy was so quickly and

Johann Wolfgang von Goethe

Goethe thought that his work on color theory (which most modern scientists consider useless) exceeded in importance all his poems and plays.

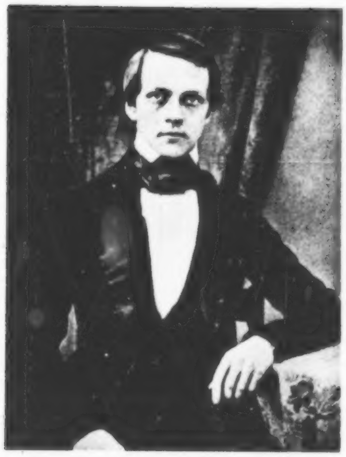
Friedrich von Schelling

successfully put to use in physics that its philosophical origins were soon forgotten.

See "The Seven Images of Science" in Project Physics Reader 3.

In their day-to-day work in laboratories and studies, many scientists stay close to experiments and mathematical calculations. However when truly major advances in science are made, such as the establishment of the law of conservation of energy, philosophical speculation often plays an important role. Creative work in science, as in any field, requires daring and imaginative minds.

Mayer and Joule were only two of at least a dozen people who, between 1832 and 1854, proposed in some form the idea that energy is conserved. Some expressed the idea obscurely; others, quite clearly. Some arrived at their belief primarily from philosophical considerations; others from a practical concern with engines and machines, or from laboratory investigations of energy conversion; still others from a combination of factors.



Hermann von Helmholtz

Helmholtz's paper, "Zur Erhaltung der Kraft," was tightly reasoned and mathematically sophisticated. It related the law of conservation of energy to the established principles of Newtonian mechanics and thereby helped make the law scientifically respectable.

The wide acceptance of the law of conservation of energy owes much to a paper published in 1847 (two years before Joule published the results of his most precise experiments) by the young German physician and physicist, Hermann von Helmholtz and entitled "On the Conservation of Force." In the paper Helmholtz boldly asserted the idea that others were vaguely expressing; namely, "that it is impossible to create a lasting motive force out of nothing." The idea was even more clearly expressed many years later in one of Helmholtz's popular lectures:

> We arrive at the conclusion that Nature as a whole possesses a store of force [energy] which cannot in any way be either increased or diminished, and that, therefore, the quantity of force [energy] in Nature is just as eternal and unalterable as the quantity of matter. Expressed in this form, I have named the general law 'The Principle of the Conservation of force [energy].'

Any machine or engine that does work—provides energy—can do so only by drawing from some source of energy. The machine cannot supply more energy than it obtains from the source, and when the source is depleted, the machine will stop working. Machines and engines can only transform energy; they cannot create it or destroy it.

A collection of bodies moving and colliding and exerting forces on one another makes up a system. The total kinetic energy and potential energy of the bodies in the system is the internal energy of the system. If objects outside the system do work on the bodies in the system, the internal energy of the system will increase. The internal energy will increase also if heat is transferred from the surroundings to the system. On the other hand, if the system does work on objects outside the system, or if heat is transferred from the system to its surroundings, the internal energy will decrease. Transferring heat and doing work are the two ways of changing the internal energy of the system. These ideas can be summarized by an equation. Let H stand for (heat added to system - heat lost by system); let W stand for (work done on system - work done by system); and let ΔU stand for the increase in internal energy of the system. Then

$$H + W = \Delta U. \qquad (10.10)$$

This statement of the law of conservation of energy, emphasizing the equivalence of heat and work, is called the first law of thermodynamics.

Thermodynamics is the study of the relation between heat and mechanical energy.

Since the middle of the nineteenth century the law of conservation of energy has become one of the most fundamental laws of science. We shall encounter it again and again in this course: in the study of electricity and magnetism, in the study of the structure of atoms, and in the study of nuclear physics.

See "Energy" in Project Physics Reader 3

The principle of conservation of energy has been so successful and is now so firmly believed that most physicists find it almost inconceivable that any new phenomenon will be found that will disprove it. Whenever energy seems to appear or disappear in a system, without being accounted for by changes in known forms of energy, physicists naturally prefer to assume that some unknown kind of energy is involved, rather than accept the possibility that energy is not conserved. We have already mentioned one example of this attitude: the concept of "internal energy" was introduced in order to preserve the validity of the principle of conservation of energy in the case of inelastic collisions and frictional processes. In this case, the physicist's faith in energy conservation was justified, because other evidence showed that internal energy is a useful concept, and that it changes by just the right amount to compensate for changes in external energy.

See "Scientific Cranks" in Project Physics Reader 3.

Another recent example is the "invention" of the neutrino by Wolfgang Pauli in 1933. Experiments had suggested that energy disappeared in certain nuclear reactions; but Pauli proposed that a tiny particle, the neutrino, was produced in these reactions and, unnoticed, carried off some of the energy. Physicists believed in the neutrino theory for more than twenty years even though neutrinos could not be directly found by any method. Finally, in 1956, neutrinos were detected, in an experiment using the large flow of neutrinos that comes out of a nuclear reactor. (The experiment could not have been done in 1933, since no nuclear reactors existed until nearly a decade later.) Again, the faith of physicists in the law of conservation of energy turned out to be justified.

We believe this will probably always be the case: any apparent exceptions to the law of conservation of energy will sooner or later turn out to be understandable in a way which does not force us to give up the law. At most, they may force us to postulate new forms of energy so that the law will become even more generally applicable and powerful.

The French mathematician and philosopher Henri Poincaré expressed this idea in 1903 in his book *Science and Hypothesis*.

> Since we can not give a general definition of energy, the principle of conservation of energy signifies simply that there is *something* which remains constant. Indeed, no matter what new notions future experiences will give us of the world, we are sure in advance that there will be something which will remain constant and which we shall be able to call *energy*.

Q23 The significance of German nature philosophy in the history of science is that
- a) it was the most extreme form of the mechanistic viewpoint.
- b) it was a reaction against excessive speculation.
- c) it stimulated speculation about the unity of natural forces.

Q24 Discoveries in electricity and magnetism early in the nineteenth century contributed to the discovery of the law of conservation of energy because
- a) they attracted attention to conversions of energy from one form to another.
- b) they made it possible to produce more energy at less cost.
- c) they revealed what happened to the energy that was apparently lost in steam engines.
- d) they made it possible to transmit energy over long distances.

Q25 The development of steam engines helped the discovery of the law of conservation of energy because
- a) steam engines produce a large amount of energy.
- b) the caloric theory could not explain how steam engines worked.
- c) steam engines used up so much water that other sources of energy had to be found.
- d) the concept of work was developed in order to compare the economic value of different engines.

Q26 According to the first law of thermodynamics
- a) the net heat added to a system is always conserved.
- b) the net heat added to a system always equals the net work done by the system.
- c) energy input equals energy output if the internal energy of the system does not change.
- d) the internal energy of a system is always conserved.

Q27 Both Mayer and Joule helped establish the law of conservation of energy. Compare their approaches to the question.

The ultimate source of most of the energy we use is the sun. Water warmed by the sun evaporates to form clouds from which the rain falls to replenish the rivers that drive the hydroelectric generators. Winds, that pushed Phoenician ships to the Pillars of Hercules and beyond, that made possible Columbus' voyage and Magellan's circumnavigation of the world, are produced when the sun warms parts of the earth's atmosphere.

The energy we obtain from food was once solar energy, locked into molecules in green plants by the complex process called photosynthesis, and released in our bodies by the process called respiration.

Coal and oil, still our major sources of industrial energy, are fossilized remains of plants and animals, with energy from sunlight still locked within their molecules.

Study Guide

10.1 An electron of mass 9.1×10^{-31} kg is traveling 2×10^6 meters per second toward the screen of a television set. What is its kinetic energy? How many electrons like this one would it take to make up a joule of energy?

10.2 Estimate the kinetic energy of each of the following:

a) a pitched baseball
b) a jet plane
c) a sprinter in a 100-yard dash
d) the earth in its motion around the sun

10.3 As a home experiment, hang weights on a rubber band and measure its elongation. Plot the force vs. stretch on graph paper. How could you measure the stored energy?

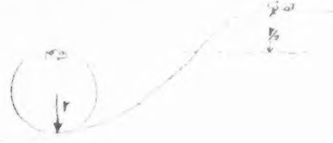
10.4 A penny has a mass of about 3.0 grams and is about 1.5 millimeters thick. You have 50 pennies which you pile one above the other.

a) How much more gravitational potential energy has the top penny than the bottom one?
b) How much more have all 50 pennies together than the bottom one?

10.5 Discuss the following statement: All the chemical energy of the gasoline used in your family automobile is used only to heat up the car, the road, and the air.

10.6 A 200-kilogram iceboat is supported by a smooth surface of a frozen lake. The wind exerts on the boat a constant force of 1000 newtons, while the boat moves 900 meters. Assume that frictional forces are negligible, and that the boat starts from rest. Find the speed after 900 meters by each of the following methods:

a) Use Newton's second law to find the acceleration of the boat. How long does it take to move 900 meters? How fast will it be moving then?
b) Find the final speed of the boat by equating the work done on it by the wind and the increase in its kinetic energy. Compare your result with your answer in (a).



10.7 The figure shows a model of a carnival "loop-the-loop." A car starting from a platform above the top of the loop coasts down and around the loop without falling off the track. Show that, to successfully traverse the loop, the car need start no higher than one-half a radius above the top of the loop. Neglect frictional forces.
HINT: The centripetal force required at the top of the loop must be greater than the weight of the car.

10.8 A cardboard tube a meter long closed at both ends contains some lead shot. It is turned end for end 300 times, each time allowing the shot to fall the length of the tube and come to rest before inverting it again. If the shot loses a negligible amount of heat to the tube, what is the increase in temperature? Try it.

10.9 An electric coffee pot holds a kilogram (about a quart) of water and is rated at 600 watts. Starting from room temperature (20°C), estimate how long it will take the water to reach boiling temperature. What assumptions have you made?

10.10 Show that if a constant propelling force F keeps a vehicle moving at a constant speed v, the power required is equal to Fv.

10.11 The Queen Mary, one of Britain's largest steamships, has been retired to a marine museum on our west coast after completing 1,000 crossings of the Atlantic. Her mass is 81,000 tons (75 million kilograms) and her maximum engine power of 234,000 horsepower (174 million watts) gives her a maximum speed of 30.63 knots (16 meters per second).

a) What is her kinetic energy at full speed?
b) What constant force would be required to stop her from full speed within 10 nautical miles (20,000 meters)?
c) What power would be required to keep her going at full speed against this force?
d) Assume that at maximum speed all the power output of her engines goes into overcoming water drag. If the engines are suddenly stopped, how far will the ship coast before stopping? (Assume water drag is constant.)
e) The assumptions made in (d) are not valid for the following reasons:
    1) Only about 60% of the power delivered to the propellor shafts results in a forward thrust to the ship; the rest results in turbulence, eventually warming the water.
    2) Water drag is less for lower speed.
    3) If the propellors are not free-wheeling, they add an increased drag. Which of the above reasons tend to increase, which to decrease the coasting distance?
f) Explain why tugboats are important for docking big ships.

10.12 Consider the following hypothetical values for a paddle-wheel experiment like Joule's: a 1 kilogram weight descends through a distance of 1 meter, turning a paddle-wheel immersed in 5 kilograms of water.

a) About how many times must the weight be allowed to fall in order that the temperature of the water will be increased by 1/2 Celsius degree?

b) List ways you could modify the experiment so that the same temperature rise would be produced with fewer falls of the weight? (There are at least four possible ways.)

10.13 On his honeymoon in Switzerland, Joule attempted to measure the difference in temperature between the top and the bottom of a waterfall. Assuming that the amount of heat produced at the bottom is equal to the decrease in gravitational potential energy, calculate roughly the temperature difference you would expect to observe between the top and bottom of a waterfall about 50 meters high, such as Niagara Falls.

10.14 Devise an experiment to measure the power output of

a) a man riding a bicycle.

b) a motorcycle.

c) an electric motor.

10.15 When a person's food intake supplies less energy than he uses, he starts "burning" his own stored fat for energy. The oxidation of a pound of animal fat provides about 4,300 kilocalories of energy. Suppose that on your present diet of 4,000 kilocalories a day you neither gain nor lose weight. If you cut your diet to 3,000 kilocalories and maintain your present physical activity, about how many weeks would it take to reduce your mass by 5 pounds?

10.16 About how many kilograms of boiled potatoes would you have to eat to supply the energy for a half-hour of swimming? Assume that your body is 20% efficient.

10.17 In order to engage in normal light work, an average native of India needs about 1,950 kilocalories of food energy a day, whereas an average West European needs about 3,000 kilocalories a day. Explain how each of the following statements makes the difference in energy need understandable.

a) The average adult Indian weighs about 110 pounds; the average adult West European weighs about 150 pounds.

b) India has a warm climate.

c) The age distribution is different in India.

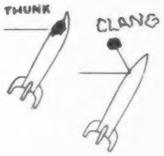
10.18 Show how the conservation laws for energy and momentum apply to a rocket lifting off.

10.19 In each of the following, trace the chain of energy transformations from the sun to the final form of energy:

a) A pot of water is boiled on an electric stove.

b) An automobile accelerates from rest on a level road, climbs a hill at constant speed, and comes to stop at a traffic light.

c) A windmill in Holland pumps water out of a flooded field.



10.20 The two identical space vehicles shown here were drifting through interstellar space. Each was struck by a 10-kilogram meteor traveling at 100 m/sec. Which rocket was knocked further off course? Explain.

10.21 A 2-gram bullet is shot into a tree stump. It enters at a speed of 300 m/sec and comes to rest after having penetrated 5 cm in a straight line. Compute the average force on the bullet during the impact, and the work done.

10.22 In the Prologue to Unit 1 of this course, it was stated that Fermi used materials containing hydrogen to slow down neutrons. Explain why collisions with light atoms would be more effective in slowing down neutrons than collisions with heavy atoms.

10.23 The actual cost of moving furniture and individuals various distances is shown in the following tables. Using these tables, discuss the statement: "the cost of moving is approximately proportional to the amount of work that has to be done on it, using the physicist's definition of work."

Truck Transportation (1965)

Weight	Moving rates (including pickup & delivery) from Boston to:		
	Chicago (967 miles)	Denver (1969 miles)	Los Angeles (2994 miles)
100 lbs	$ 18.40	$ 24.00	$ 27.25
500	92.00	120.00	136.25
1000	128.50	185.50	220.50
2000	225.00	336.00	406.00
4000	384.00	606.00	748.00
6000	576.00	909.00	1122.00

Air Cargo Transportation (1965)

Weight	Moving rates (including pickup & delivery) from Boston to:		
	Chicago	Denver	Los Angeles
100 lbs	$ 13.95	$ 25.57	$ 29.85
500	70.00	127.50	149.25
1000	129.00	216.50	283.50
2000	248.00	413.00	527.00
4000	480.00	796.00	1024.00
6000	708.00	1164.00	1255.00

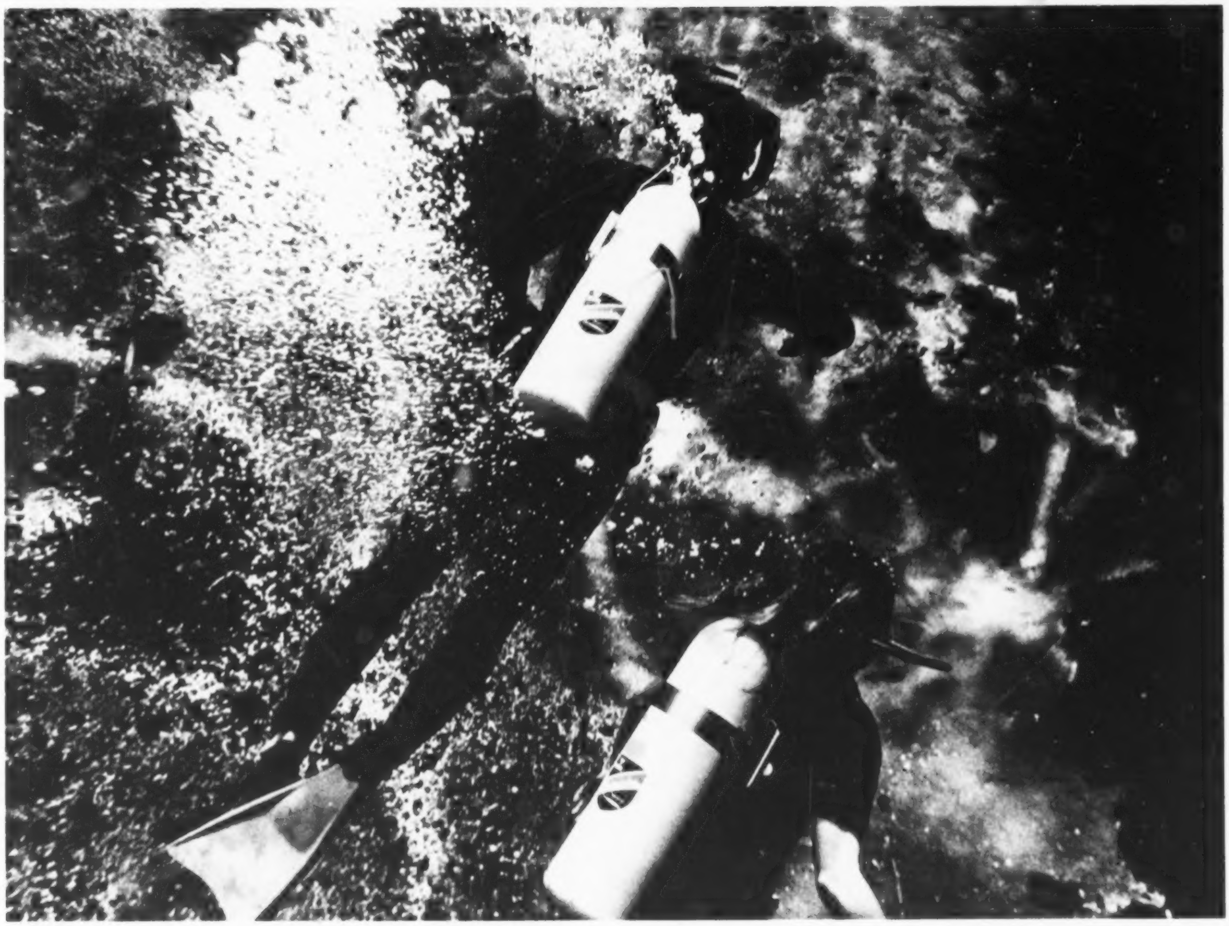
Personal Transportation (1965)

	One way fare from Boston to:		
	Chicago	Denver	Los Angeles
Bus	$33.75	$58.40	$88.75
Train	$47.85	$79.31	$115.24
Airplane (jet coach)	$53.39	$107.36	$167.58

Chapter 11 The Kinetic Theory of Gases

Section		Page
11.1	An overview	77
11.2	A model for the gaseous state	79
11.3	The speeds of molecules	81
11.4	The sizes of molecules	85
11.5	Predicting the behavior of gases from the kinetic theory (Special topic)	91
11.6	The second law of thermodynamics and the dissipation of energy	98
11.7	Maxwell's demon and the statistical view of the second law of thermodynamics	101
11.8	Time's arrow and the recurrence paradox	103

Bubbles of gas from high-pressure tanks expand as the pressure decreases on the way to the surface.



11.1 An overview. During the 1840's many scientists came to the conclusion that heat is not a substance but a form of energy which can be converted into other forms (see Chapter 10). Two of these scientists, James Prescott Joule and Rudolf Clausius, then took what seemed to be the next logical step: they assumed that since heat can be changed into mechanical energy and mechanical energy can be changed into heat, then heat itself might be a form of mechanical energy. They proposed, as a first approximation, that "heat energy" is simply the kinetic energy of atoms and molecules.

Since nineteenth-century scientists could not observe the motions of individual molecules, they could not check directly the hypothesis that heat is molecular kinetic energy. Instead, they first had to work out some mathematical predictions from the hypothesis, and then try to test these predictions by experiment. For theoretical reasons which we will explain below, it is easiest to test such hypotheses by working with the properties of gases, and therefore this chapter will deal primarily with what is called the kinetic theory of gases.

The development of the kinetic theory of gases in the nineteenth-century led to the last major triumph of Newtonian mechanics. By using a simple theoretical model of a gas, and applying Newton's laws of motion, scientists could deduce equations which related observable properties of gases—such as pressure, density and temperature—to the sizes and speeds of molecules. With these equations, kinetic theorists could

(1) explain the known relations between observable properties of gases, such as "Boyle's law";

(2) predict new relations, such as the fact that the viscosity of a gas increases with temperature but is independent of density;

(3) infer the sizes and speeds of the molecules.

Thus the success of kinetic theory indicated that Newtonian mechanics could be used to investigate the properties of matter as observed in the laboratory, and for the first time provided a reliable way of estimating the properties of individual molecules.

By applying Newtonian mechanics to a mechanical model of gases, however, the kinetic theorists made another discovery: they found physical situations in which Newtonian mechanics is not valid. According to kinetic theory, the energy of a molecule should be equally shared by the motions of all the atoms inside the molecule. But the properties of gases

Recall from chemistry that molecules may contain several atoms.

deduced from this "equal sharing" principle were definitely in disagreement with experiment. Newtonian mechanics could be applied successfully to the motions and collisions of molecules among each other in a gas, but not to the motions of atoms inside molecules. It was not until the twentieth century that an adequate theory of the behavior of atoms — quantum mechanics — was developed (see Unit 5).

Kinetic theory also revealed another apparent contradiction between Newtonian mechanics and observable properties of matter: this is the problem of "irreversibility." An inelastic collision is an example of an irreversible process; other examples are the mixing of two gases, or scrambling an egg. Can irreversible processes be described by a theory based on Newtonian mechanics, or do they involve some new fundamental law of nature? In discussing this problem from the viewpoint of kinetic theory, we will see how the concept of "randomness" was introduced into physics.

Modern physicists do not take seriously the "billiard ball" model of gas molecules—nor did nineteenth-century physicists, for that matter. The simple postulates of the model have to be modified in many respects in order to get a theory that agrees well with experimental data. Nevertheless, physicists are still very fond of the kinetic theory, and often present it as an example of how a physical theory should be developed. Perhaps this is only nostalgia for the old mechanistic style of explanation which has had to be abandoned in other areas of physics. But as an example of an ideal type of theory, kinetic theory has exerted a powerful influence on physics research and teaching. In Sec. 11.5, therefore, you will find one of the mathematical derivations from a mechanical model which is used in kinetic theory. This derivation is given, not to be studied in detail, but as an illustration of the kind of mathematical reasoning based on mechanical models which physicists have found to be useful in understanding nature.

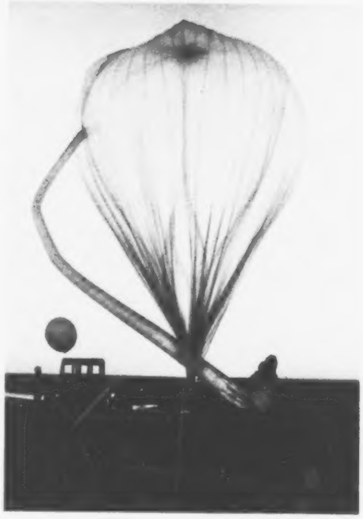
Q1 Nineteenth-century kinetic theorists assumed that heat is

a) a fluid.
b) molecular.
c) the kinetic energy of molecules.
d) made of molecules.

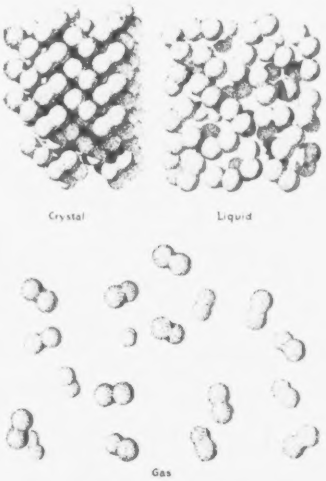
Q2 In the kinetic theory of gases, it is assumed that Newton's laws of motion (do, do not) apply to the motion and collisions of molecules.

Q3 True or false: In the twentieth century Newtonian mechanics was found to be applicable not only to molecules but also to the atoms inside molecules.

11.2 *A model for the gaseous state*. What is the difference between a gas and a liquid or solid? We know by observation that a liquid or solid has a definite volume. A gas, on the other hand, will expand to fill any container, or, if unconfined, will go off in all directions. We can also find out easily that gases have low densities compared to liquids and solids. According to the kinetic theory, the molecules in a gas are moving freely through empty space most of the time, occasionally colliding with each other or with the walls of their container. Furthermore, we assume that the forces between molecules act only at very short distances, whereas the molecules are usually far apart from each other. Therefore gas molecules, in this model, are considered "free" except during collisions. In liquids, on the other hand, the molecules are so close together that these forces keep them from flying apart. In solids, the molecules are usually even closer together, and the forces between them keep them in a definite orderly arrangement.


Weather balloon of U.S. Air Force.

In the nineteenth century, little was known about the forces between molecules, so it was natural to apply the kinetic theory first to gases, where these forces should have little effect if the theory is right. Physicists therefore adopted a simple *model* of gases, in which the molecules are considered to behave like miniature billiard balls—that is, tiny spheres which exert no forces at all on each other except when they collide. All the collisions of these spheres are assumed to be elastic, so that the total kinetic energy of two spheres is the same before and after they collide. The total volume occupied by all these spheres is assumed to be very small compared to the total volume of the container.



Sketch of the relative spacing of atoms in three states of matter.

from *General Chemistry*, 2nd edition, by Linus Pauling, W.H. Freeman and Company, © 1953.

Note that the word "model" is used in two different senses in science. In Chapter 10, we mentioned the model of Newcomen's engine which James Watt was given to repair. That was a *working model* which actually could do work, although it was much smaller than the original engine, and some of its parts were made of different materials. We are now discussing a *theoretical model* of a gas; this model exists only in our imagination. Like the points, lines, triangles and spheres which are studied in geometry, this theoretical model can be discussed mathematically, and the results may or may not be useful to us when we try to understand the real world of experience. In order to emphasize the fact that the model is only a theoretical one, we will use the word "particle" instead of atom or molecule. Atoms and molecules exist and have definite properties that can be determined in the laboratory. Particles are imaginary objects, such as perfectly elastic spheres, whose properties as defined in our theoretical model may or may not be similar to those of atoms and molecules. We will develop the model in such a way that it can be used to explain as well as possible the particular properties of gases in which we are interested.

The word "gas" was originally derived from the Greek work "chaos"; it was first used by the Belgian chemist Joan Baptista van Helmont (1580-1644).

Our model of a gas consists of a large number of very small particles in rapid, disordered motion. We now know enough about gases to say more precisely how we should interpret this phrase if we want to have a useful model. "A large number" here means something like a billion billion (10^{18}) or more. "Very small" means about a hundred-millionth of a centimeter (10^{-8} cm, or 10^{-10} meters) in diameter. "Rapid motion" means a speed of a few hundred meters per second. In the following sections we will see how the model was used by kinetic theorists to arrive at these estimates.

What do we mean by "disordered"? In the nineteenth century, kinetic theorists assumed that if they could follow the motion of an individual molecule they would see it move in a definite way determined by Newton's laws of motion. However, it would be very difficult to calculate the properties of the kinetic theory gas model if we had to apply Newton's laws to a billion billion particles. We don't know in what direction any given particle will be moving. Furthermore, even though we can estimate the average speed of the particles, different particles will have different speeds and each particle will frequently change its speed. Therefore from our viewpoint the motions of the individual particles are disordered and unpredictable, and it is useful to assume that these motions are random even though we believe that they are really determined by the laws of motion.

See "The Law" and "Randomness and the Twentieth Century" in Project Physics Reader 3

The idea of randomness originally entered physics through the kinetic theory of gases. The disadvantage of having to consider the motions of a large number of particles actually becomes an advantage as soon as one assumes that the particles move randomly. You can see why this is so by thinking about the results of flipping a coin. It would be very hard to predict how a single coin would behave, but you can confidently predict that flipping a million coins will give about 50 percent heads and 50 percent tails. Similarly, it is a safe bet that out of a billion billion molecules about as many will be moving in one direction as in another. It is also reasonable to expect that the particles are equally likely to be found anywhere inside the container, even though we don't

On page 86 you will find a more detailed discussion of the idea of random fluctuations.

know where a given particle is at any particular time.

In summary: we are going to discuss the properties of a mechanical model for gases. The model consists of a large number of very small particles in rapid disordered motion. The particles move freely most of the time, exerting forces on each other only when they collide. The model is designed to represent the structure of real gases in some ways, but it is simplified in order to make calculations possible. By comparing the results of these calculations with the properties of gases it is possible to estimate the speeds and sizes of molecules, assuming that the model itself is a reasonably good description of gases.

See "On the kinetic theory of gases" and "The Law of Disorder" in Project Physics Reader 3

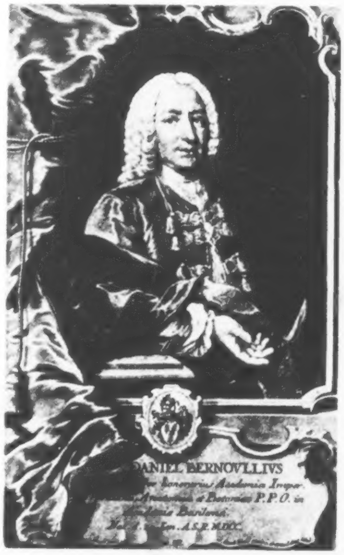
Q4 Molecules exert forces on one another

a) only when the molecules are far apart.
b) only when the molecules are close together.
c) all the time.
d) never.

Q5 Why was the kinetic theory first applied to gases rather than to liquids or solids?

11.3 The speeds of molecules. The basic idea of the kinetic theory—that heat is related to the kinetic energy of molecular motion—had been frequently suggested in the seventeenth century. In 1738, the Swiss mathematician Daniel Bernoulli showed how the kinetic theory could be used to explain a well-known property of gases: pressure is proportional to density (Boyle's law). Bernoulli assumed that the pressure of a gas is simply a result of the impacts of individual molecules striking the wall of the container. If the density were doubled, there would be twice as many molecules striking the wall per second, and hence twice the pressure.

Pressure is defined as the perpendicular force on a surface divided by the area of the surface.



Daniel Bernoulli

Bernoulli also proved mathematically that the pressure would be proportional to the square of the speed of the molecules. Later scientists used this proof to establish relations between pressure, molecular speed and temperature (see Sec. 11.5). However, in Bernoulli's time, the modern temperature scales had not been established, and Bernoulli did not have enough experimental data to develop this part of his theory.

Although Bernoulli's theory seems to have been the first step leading to the modern kinetic theory of gases, it was generally ignored by other scientists in the eighteenth century. One reason for this was that Newton had proposed a different theory in his Principia (1687). Newton showed that Boyle's law could be explained by assuming that a gas is composed of particles at rest that exert repulsive forces on neighboring particles, if these forces are inversely proportional to the distance between particles. Although Newton did not claim that he had proved that gases really are composed of such repelling particles, other scientists were so impressed by Newton's discoveries in other areas of physics that they assumed his theory of gas pressure must also be right.

John Herapath

The kinetic theory of gases was proposed again in 1820 by an English physicist, John Herapath. Herapath rediscovered Bernoulli's results on the relation between pressure and density of a gas and the speeds of the particles. In modern symbols, we can express these results by the simple equation

$$P = \frac{1}{3} Dv^2 \qquad (11.1)$$

where P = pressure which the gas exerts on the container, D = density (mass/volume) and v = average speed. (This equation is important still, and will be derived in Sec. 11.5) Since we can determine the pressure and density of a gas by experiment, we can use this result to calculate the average speed of the molecules. Herapath did this, and found that the result was fairly close to the speed of sound in the gas, about 330 meters per second for air.

Herapath's calculation of the speed of an air molecule (first published in 1836) was an important event in the history of science, but it was ignored by most other scientists. Herapath's earlier work on the kinetic theory had been rejected for publication by the Royal Society of London, and despite a long and bitter battle (including letters to the Editor of the Times of London) Herapath had not succeeded in getting any recognition for his theory.

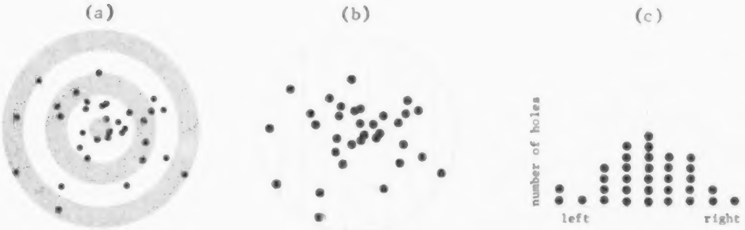
James Prescott Joule did see the value of Herapath's work, and in 1848 he read a paper to the Manchester Literary and Philosophical Society in which he tried to revive the kinetic theory. Joule showed how the speed of a hydrogen molecule could be computed (using Herapath's theory), and obtained a value of 2000 meters per second at 0°C, the freezing temperature of water.

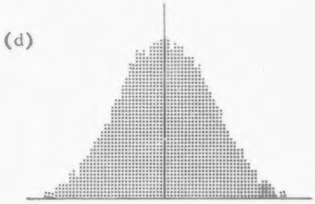
Joule's paper also was ignored by other scientists, because physicists do not generally look in the publications of a "literary and philosophical society" for important papers. But once the equivalence of heat and mechanical energy was generally accepted, it was not long before other physicists independently started to work out the consequences of the assumption that heat is molecular energy. Rudolf Clausius in Germany published a paper in 1856 on "The Nature of the Motion we call Heat," which established the basic principles of kinetic theory in almost the form we accept them today. Soon afterwards, James Clerk Maxwell in Britain, and Ludwig Boltzmann in Austria, started to work out the full mathematical details of the theory.

The Maxwell velocity distribution. Not all molecules in a gas have the same speed. In 1859 Maxwell suggested that the speeds of molecules in a gas could be described by a statistical law: most molecules have speeds not very far from the average speed, but a few have much lower speeds and a few have much higher speeds.

Maxwell's law of distribution of molecular speeds can be illustrated by the following analogy: if a marksman shoots a gun at a target many times, some of the bullets will probably hit the bullseye, but others will miss by smaller or larger amounts. Suppose we count the number of bullets that hit the target at various distances to the left and right of the bullseye, and make a graph showing the number of bullets at these distances.

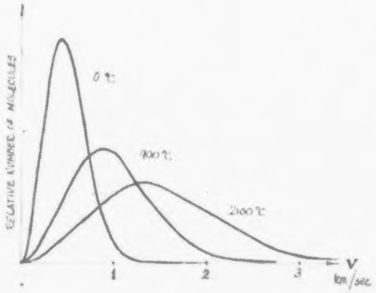
TARGET PRACTICE EXPERIMENT
(a) Scatter of holes in target; (b) target marked off in distance intervals left and right of center; (c) frequency distribution of holes left and right of center. If, as in (d), there is a very large number of cases, such distribution often closely approximate the mathematical curve called the "normal distribution."



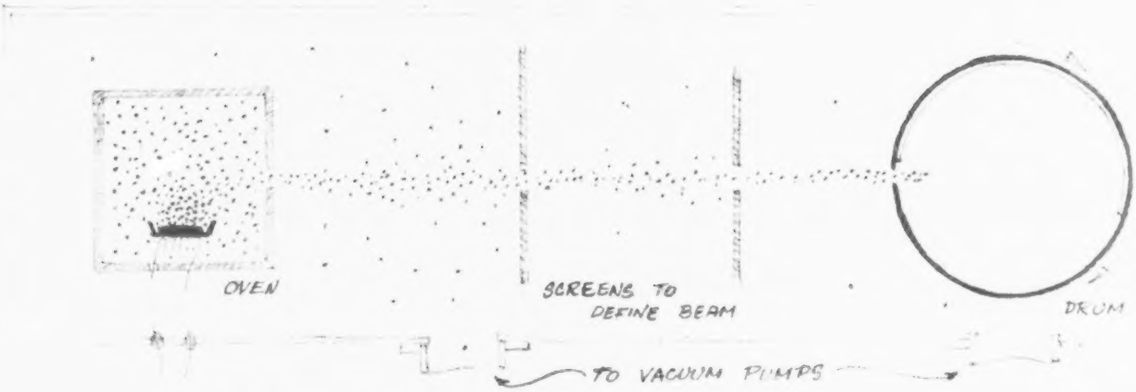


The graph showing the distribution of misses illustrates a general principle of statistics, the law of normal distribution: if any quantity varies randomly about an average value, then the graph showing the frequencies of deviations from the average will look something like the one shown in the margin. There will be a peak at about the average value and a sharp decline on either side. A similar "bell shaped curve," as it is frequently called, describes the distribution of many kinds of physical measurements. During the nineteenth century, for example, statisticians found that the normal distribution law applies even to large collections of people, as in the distribution of human heights.

Maxwell's distribution law for speeds is shown in the margin. Note that the peak shifts to higher speeds as the temperature increases. The tail of the curve is much longer on the right (high speeds) than on the left (low speeds), since no molecules can have less than zero speed.

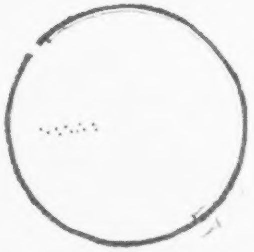


Maxwell's distribution of molecular speeds for 3 different temperatures.

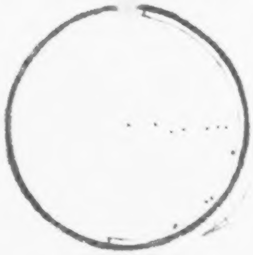


Direct Measurement of Molecular Speeds

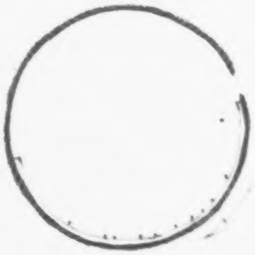
A narrow beam of molecules is formed by letting a hot gas pass through a series of slits. In order to keep the beam from spreading out, collisions with randomly moving molecules must be avoided; therefore, the source of gas and the slits are housed in a highly evacuated chamber. The molecules are then allowed to pass through a slit in the side of a cylindrical drum which can be spun very rapidly. The general scheme is shown above.



As the drum rotates, the slit moves out of the beam of molecules so that no more molecules can enter until the drum has rotated through a whole revolution. Meanwhile the molecules in the drum continue moving to the right, some moving fast and some moving slowly.

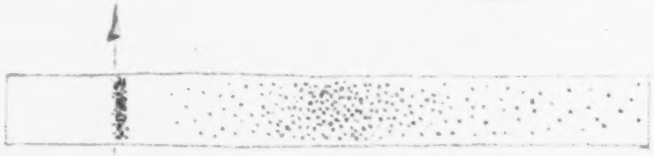


Fastened to the inside of the drum is a sensitive film which acts as a detector; any molecule striking the film leaves a mark. The faster molecules strike the film first, before the drum has rotated very far.

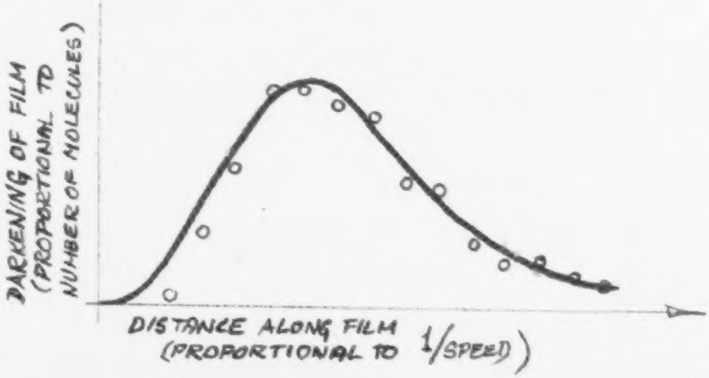


The slower molecules hit the film later, after the drum has rotated farther. In other words, molecules of different speeds strike different parts of the film. The darkness of the film at any point is proportional to the number of molecules which hit it there. Measurement of the darkening of the film shows the distribution of molecular speeds.

The speckled strip at the right represents the unrolled film, showing the impact position of molecules over many revolutions of the drum. The heavy band indicates where the beam struck the film before the drum started rotating.



A comparison of the experimental results with those predicted from theory is shown in the graph. The dots show the experimental results and the solid line represents the predictions from the kinetic theory.



What reason do we have for thinking that Maxwell's distribution law really applies to molecular speeds? Several successful predictions based on this law gave indirect support to it, but it was not until the 1920's that a direct experimental check was possible. Otto Stern in Germany, and later Zartmann in the United States, devised a method for measuring the speeds in a beam of molecules (see the illustration of Zartmann's method on the preceding page). Stern, Zartmann and others found that molecular speeds are indeed distributed in accordance with Maxwell's law.

Q6 In the kinetic theory of gases it is assumed that the pressure of a gas is due to

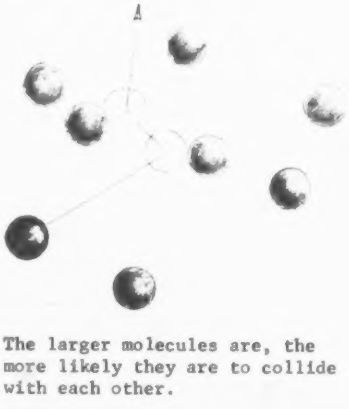
a) gas molecules colliding with one another.
b) gas molecules colliding with the walls of the container.
c) repulsive forces exerted by molecules on each other.

Q7 The average speed of molecules in a gas can be calculated if we know

a) the pressure of the gas and the volume of the container.
b) the mass of the gas and the volume of the container.
c) the pressure of the gas and its density.

11.4 The sizes of molecules. Is it reasonable to suppose that gases are composed of molecules that move at speeds of several hundred meters per second? In 1857, a Dutch meteorologist, Christian Buys-Ballot, pointed out that if this were really true, one would expect gases to diffuse and mix with each other very rapidly. But anyone who has studied chemistry knows that if hydrogen sulfide or chlorine is generated at one end of a large room, it may be several minutes before it is noticed at the other end. Yet, according to the kinetic-theory calculations we mentioned in the last section, each of the gas molecules should have crossed the room hundreds of times by then. Something must be wrong with our model.

Rudolf Clausius realized that this discrepancy was a valid objection to his own version of the kinetic theory. In his paper published in 1856, he had assumed that the size of the particles is so small that they can be treated like mathematical points. If this were true, particles would almost never collide with each other. However, in order to account for the gas properties pointed out by Buys-Ballot—that is, the slowness of diffusion and mixing—Clausius decided to change the model. He thought it was likely that in real gases, the molecules are not mathematical points but have a finite size. He realized that if the model were made more realistic by assuming that the particles have finite size and can therefore collide with each other, it would then be possible to explain why gases do not diffuse rapidly.



The larger molecules are, the more likely they are to collide with each other.

Averages and Fluctuations

Molecules are too small, too numerous, and too fast for us to measure the speed of any one molecule, or its kinetic energy, or how far it moves before colliding with another molecule. For this reason the kinetic theory of gases concerns itself with making predictions about average values. The theory enables us to predict quite precisely the average speed of the molecules in a sample of gas, or the average kinetic energy, or the average distance the molecules move between collisions.

The average of a small number of cases cannot be predicted very well. Although the average height of adult American men is 5'9½", it would be very unlikely that the average height of any particular group of 10 men would be that value. However, statistical predictions can be very precise for very large sets of values. The average height of all adult men in Ohio would be very close to 5'9½". The precision in predicting average values for very large samples is what makes the kinetic theory of gases so successful, for molecules are very numerous indeed.

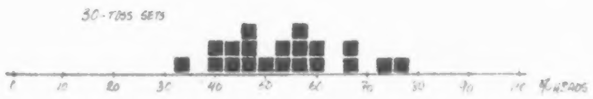
A simple example of a statistical prediction is the statement that if a coin is tossed many times, it will land "heads" 50 percent of the time and "tails" 50 percent of the time. For small sets of tosses, there will be many "fluctuations" away from the predicted average of 50% heads. The first chart at the right shows the percentage of heads in twenty sets of 30 tosses each. Because there are more ways that 30 tosses can split 15-15 than split any other way, 50% is the most probable value for every set. But often the split is not 15-15.

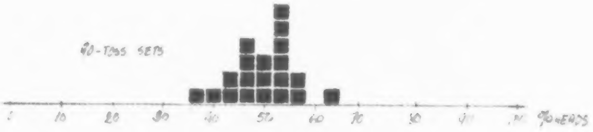
The next chart shows the percentage of heads in each of twenty 90-toss sets. There are still fluctuations from the most probable 45-45 split, but the fluctuations are generally smaller compared to the total number of tosses. Large fluctuations from 50% are less common than for the smaller 30-toss sets.

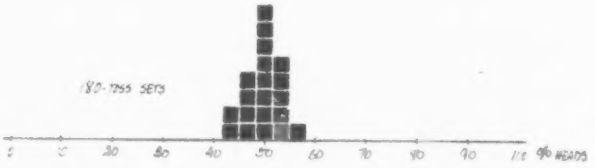
The last chart shows the percentage of heads in each of twenty 180-toss sets. There are certainly fluctuations, as there always will be, but they are generally smaller compared to the total number of tosses in a set. Large fluctuations from 50% are still less common.

Statistical theory shows that the average fluctuation from 50% for sets of tosses shrinks in proportion to the square root of the number of tosses. We can use this rule to compare the average fluctuation for sets of, say, 100,000,000 tosses with the average fluctuation for sets of 100 tosses: since the 100,000,000-toss sets have 1,000,000 times as many tosses as the 100-toss sets, their average fluctuation in percentage of heads should be 1,000 times smaller!

These same principles hold for fluctuations from average values of any randomly-distributed quantities, such as molecular speed, or distance between collisions. Since even a thimble-full of air contains about a quintillion (10^{18}) molecules, large fluctuations in observable gas behavior are extremely unlikely.



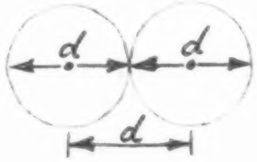




Even though an individual molecule has an instantaneous speed of several hundred meters per second, it changes its direction of motion every time it collides with another molecule. Consequently it doesn't get very far away from its starting point.

Clausius now was faced with the dilemma that plagues every theoretical physicist. If a simple model is modified to explain more observable properties, it will then be more complicated, and one usually will not be able to deduce the predictions of the model from its basic assumptions without making some approximations. If the theoretical predictions don't agree with the experimental data, one doesn't know whether this is because one of the assumptions of the model is wrong, or because some error was introduced by the approximations made in doing the calculation from the model. (This situation has been somewhat improved in the twentieth century by the availability of fast electronic computers, but the problem is still a serious one.) The development of a theory often involved a compromise between two criteria: adequate explanation of the data and mathematical convenience.

Clausius found a temporary solution by making only a small change in the model: he assumed that the particles are not points but spheres of diameter d. Two particles will collide with each other if their centers come within a distance d; all collisions are still assumed to be perfectly elastic.



The relation between mean free path, collision probabilty and molecular size will be investigated in the experiment which goes along with this chapter. Mathematical derivations will also be found in the discussion of this experiment in the Student Handbook.

Using his new model, Clausius proved mathematically that the "mean free path" of a particle (defined as the average distance it travels between collisions) is inversely proportional to the square of the diameter of the particles. The probability that a particle will collide with another one is proportional to the cross-sectional area of a particle, and this area is proportional to the square of its diameter. The bigger the particle, the more likely it is to collide with others, and the shorter its mean free path will be.

Within a few years it became clear that the new model was a great improvement over the old one. It turned out that other properties of gases also depend on the size of the molecules, and by combining data on several such properties it was possible to work backwards and determine fairly accurate values for molecular sizes.

It was first necessary to find a precise theoretical relation between molecular size and a measurable property of gases. This was done by Maxwell soon after Clausius' paper on the

See "James Clerk Maxwell" in *Project Physics Reader 3*

mean free path was published. Fortunately, a record of some of Maxwell's earliest thoughts on kinetic theory has been preserved in his correspondence. On May 30, 1859, Maxwell wrote a letter to Sir George Gabriel Stokes, a prominent mathematical physicist and expert on the properties of fluids:

> I saw in the Philosophical Magazine of February '59, a paper by Clausius on the 'mean length of path of a particle of air or gas between consecutive collisions,'.... I thought that it might be worth while examining the hypothesis of free particles acting by impact and comparing it with phenomena which seem to depend on this 'mean path.' I have therefore begun at the beginning and drawn up the theory of the motions and collisions of free particles acting only by impact, applying it to internal friction [viscosity] of gases, diffusion of gases, and conduction of heat through a gas...
>
> ...I do not know how far such speculations may be found to agree with facts, even if they do not it is well to know that Clausius' (or rather Herapath's) theory is wrong and at any rate as I found myself able and willing to deduce the laws of motion of systems of particles acting on each other only by impact, I have done so as an exercise in mechanics. One curious result is that μ [the viscosity coefficient] is independent of density... This is certainly very unexpected, that the friction should be as great in a rare as in a dense gas. The reason is, that in the rare gas the mean path is greater, so that the frictional action extends to greater distances.
>
> Have you the means of refuting this result of the hypothesis?

Notice that Maxwell seems intrigued by the mathematical properties of the *model*. Yet he expects that the only contribution that his calculations will make to science may be to refute the theory by showing that it leads to predictions that disagree with experiment.

Cold honey.

Maxwell's calculations showed that the viscosity of a gas should be proportional to the mass and average speed of the individual molecules, and inversely proportional to the cross-section area of each molecule:

$$\text{viscosity} \propto \frac{mv}{A}.$$

According to kinetic theory, the absolute temperature of the gas is proportional to the square of the average speed of the molecules (Sec. 11.5). Maxwell's formula therefore indicates that the viscosity of a gas should *increase* with the temperature; it should be proportional to the square root of the absolute temperature. This would be markedly different from the familiar behavior of liquids, whose viscosity *decreases* with temperature.

Stokes' reply to Maxwell's letter has been lost, but we can guess that he told Maxwell that the existing experimental data (on the viscosity of gases at different temperatures and densities) were inadequate to test the kinetic-theory prediction. At the time, no one had any reason to think that gases would behave differently from liquids in this respect. The need to make careful measurements of gas viscosity did not arise until Maxwell showed that such measurements would be of great theoretical significance in testing the kinetic theory of gases.

Although his fame rests mainly on his theoretical discoveries, Maxwell was quite competent in doing experiments, and he decided to make his own measurements of the viscosity of air. He used a laboratory in his own house in London; one can imagine the puzzlement of his neighbors who could look in the window:

> For some days a large fire was kept in the room, though it was in the midst of very hot weather. Kettles were kept on the fire, and large quantities of steam allowed to flow into the room. Mrs. Maxwell acted as stoker, which was very exhausting work when maintained for several consecutive hours. After this the room was kept cool, for subsequent experiments, by the employment of a considerable amount of ice.

Maxwell had already published his prediction about gas viscosity, along with several other mathematical theorems deduced from the assumptions of the model, before he did these experiments. It was therefore a pleasant surprise when he found that the prediction was right after all. The viscosity of a gas does increase with temperature, and does not change at all over a very large range of densities. This success not only made Maxwell himself a firm believer in the correctness of the kinetic theory, but also helped to convert many other scientists who had been somewhat skeptical.

Now that a definite relation had been established between an observable property of gases—viscosity—and the size of molecules, it was possible to use experimental data to obtain information about molecules. However, Maxwell's theory related the viscosity not only to the cross-section area of a single molecule, but also to its mass. The mass of a molecule was still an unknown quantity, although one could measure the total mass of all N molecules in the gas. It was still necessary to estimate the value of N, the number of molecules in the gas.

In 1865, the Austrian physicist Josef Loschmidt made the first quantitative estimate of molecular size from kinetic theory by combining viscosity measurements with data on the comparative volumes of liquids and gases. Loschmidt reasoned

that molecules are probably closely packed together in the liquid state, so that the volume of a given mass of liquid is approximately the same as the total volume of all the molecules in the liquid. The volume of the liquid can therefore be used to calculate the product Nd^3, and measurements of the same substance in the gaseous state can be used to calculate Nd^2. It was then possible to compute N and d separately.

Loschmidt used this method to estimate the diameter of an air molecule. He obtained a value of about a millionth of a millimeter (or 10^{-9} meters). This is about 4 times as large as modern values, but it is amazingly accurate considering the fact that before 1865 no one knew whether a molecule was bigger than a thousandth of a millimeter or smaller than a trillionth of a millimeter. In fact, as Lord Kelvin remarked five years later,

> The idea of an atom has been so constantly associated with incredible assumptions of infinite strength, absolute rigidity, mystical actions at a distance and indivisibility, that chemists and many other reasonable naturalists of modern times, losing all patience with it, have dismissed it to the realms of metaphysics, and made it smaller than 'anything we can conceive.'

Kelvin showed that other methods could also be used to estimate the size of atoms. None of these methods gave results as reliable as did the kinetic theory, but it was encouraging that they all led to the same order of magnitude (within 50 percent) for the size of a molecule.

Loschmidt's method of calculating d could also be used to calculate N, the number of molecules in a given volume of gas. The number of molecules in a cubic centimeter of gas (at 1 atmosphere pressure and 0°C) is now known as Loschmidt's number; its presently accepted value is 2.687×10^{19}.

Q8 In his kinetic-theory model Clausius assumed that the particles have a finite size, instead of being mathematical points, because

a) obviously everything must have some size.
b) it was necessary to assume a finite size in order to calculate the speed of molecules.
c) the size of a molecule was already well known before Clausius' time.
d) by assuming finite-size molecules the theory could account for the slowness of diffusion.

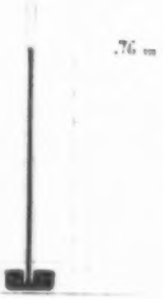
Q9 Maxwell originally thought that he could refute the kinetic theory by

a) proving that not all molecules have the same speed.
b) proving that molecules have a finite size.
c) proving that the theoretical prediction that gas viscosity is independent of density disagrees with experiment.
d) proving that gas viscosity does not decrease with increase in temperature.

11.5 Predicting the behavior of gases from the kinetic theory.

This section treats some optional special topics.

Galileo, in his Dialogues Concerning Two New Sciences (1638), noted that a vacuum pump cannot lift water more than about 34 feet (10½ meters). This fact was well known; pumps were being used to obtain drinking water from wells and to remove water from flooded mines. One important consequence of the limited ability of pumps to lift water was that some other method was needed to pump water out of deep mines. This need provided the initial stimulus for the development of steam engines (Sec. 10.5). Another consequence was that physicists in the seventeenth century became curious about why the vacuum pump worked at all, as well as about why there should be a limit to its ability to raise water.



Torricelli's barometer was a glass tube standing in a pool of mercury. The air pressure on the pool supported the column of mercury in the tube.

Air Pressure. As a result of experiments and reasoning by Torricelli (a student of Galileo), Guericke, Pascal and Robert Boyle, it was fairly well established by 1660 that the vacuum pump works because of air pressure. This pressure is sufficient to balance a column of water high enough to exert an equal pressure in the opposite direction. If mercury, which is 14 times as dense as water, is used instead, the air pressure can raise it only $\frac{1}{14}$ as far, that is, about 0.76 meter. This is a more convenient height for doing laboratory experiments and therefore much of the seventeenth-century research on air pressure was done with the mercury "barometer" designed by Torricelli.

See "The Barometer Story" in Project Physics Reader 3

It seems curious at first glance that the height of the mercury column which can be supported by air pressure does not depend on the diameter of the tube—that is, it doesn't depend on the total amount of mercury, but only on its height. To understand the reason for this we must distinguish between pressure and force. Pressure is defined as the magnitude of the force on a surface divided by the area of the surface:

$$P = F/A. \tag{11.2}$$

A large force may produce only a small pressure if it is spread over a large enough area; for example you can walk on snow without sinking in if you wear large snowshoes. On the other hand, a small force may produce a very large pressure if it is concentrated in a small area. Women wearing spike heels can ruin a wooden floor or carpet; the pressure at the place where their heels touch the floor is greater than that under an elephant's foot.

NOTICE to LADIES wearing SPIKE HEELS

Thank you

In 1661 two English scientists, Richard Towneley and Henry Power, discovered a basic relation between the pressure exerted by a gas and its density: the pressure is proportional to the density. Thus if the density of a given mass of air is doubled, by compressing it, its pressure will be twice as great. Since the density is defined as mass divided by volume, this relation can also be stated: the product pressure × volume is constant. Robert Boyle confirmed this relation by extensive experiments, and it is generally known as Boyle's law.

Boyle's law: pressure is proportional to density, if the temperature is held constant when the density changes.

Kinetic explanation of gas pressure. According to the kinetic theory of gases, the pressure of a gas is the average result of the continual impacts of many small particles against the wall of the container. It is therefore reasonable that the pressure should be proportional to the density; the greater the density, the greater the number of particles colliding with the wall. Moreover the pressure must depend on the speed of the individual particles, which determines the force exerted on the wall during each impact and the frequency of the impacts.

See "The Great Molecular Theory of Gases" in Project Physics Reader 3

We are now going to study the model of a gas described in Sec. 11.2: "a large number of very small particles in rapid disordered motion." Rather than trying to analyze the motions of particles moving in all directions with many different velocities, we fix our attention on the particles that are simply bouncing back and forth between two opposite walls of a box. Hardly any molecules in a real gas would actually move in just this way, but the basic physical factors involved in the theory can be understood fairly well with this simplified model. We will assume, then, that all the particles in our model are moving with the same speed, either right or left. In addition, we will assume that particles never hit each other, but when they hit the sides of the box they bounce off elastically, simply reversing their direction.

We are assuming here that the particles are points with zero size, so that collisions between particles can be ignored. If the particles did have finite size the results of the calculation would be slightly different, but the approximation used here is accurate enough for most purposes.

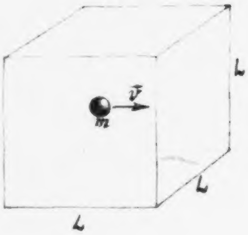
For a later part of the argument we will want to be able to move one of the walls, so we will make that wall a piston which snugly fits into the box. It can be shown from the laws of conservation of momentum and energy that when a very light particle bounces elastically off of a much more massive stationary object, very little of its kinetic energy is transferred. That is, the particle reverses direction with very little loss in speed. Bombardment by a tremendously large number of molecules would move the wall, however, so we will provide an outside force just great enough to keep the wall in place.

work = force x distance; if distance = 0, then work = 0

How large a force will these particles exert on the piston when they hit it? According to Newton's second law, the force on the particle is equal to the product of its mass times its acceleration ($\vec{F} = m\vec{a}$). As was shown in Sec. 9.4, the force can also be written as

$$\vec{F} = \frac{\Delta(m\vec{v})}{\Delta t}$$

where $\Delta(m\vec{v})$ is the change in momentum. To find the average force acting on the wall we need to find the change in momentum per second due to molecule-wall collisions. By Newton's third law the average force acting on the wall is equal and opposite to the average force acting on the molecules.



Let a single molecule of mass m move in a cubical container of volume L^3 as shown in the figure. The molecule, moving with a speed v_x, is about to collide with the right-hand wall. The momentum of the molecule just before collision is mv_x. The molecule collides elastically with the wall and rebounds with the same speed. Therefore the momentum after the collision is $m(-v_x) = -mv_x$. The change in the momentum of the molecule as a result of this collision is

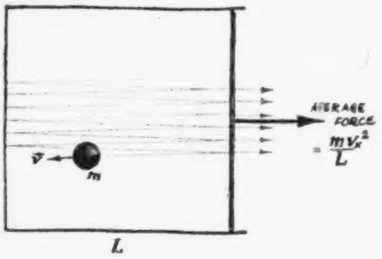
$$-mv_x - mv_x = -2mv_x.$$

The time between collisions of one molecule with the right-hand wall is the time required to cover a distance 2L at a speed of v_x; that is, $2L/v_x$. If $2L/v_x$ = the time between collisions, then $v_x/2L$ = the number of collisions per second. Thus, the change in momentum per second is given by

Note that all the vectors considered in this derivation have only two possible directions: to the right or to the left. We can therefore indicate direction by using + and - signs respectively.

$$\begin{pmatrix}\text{change in momentum}\\ \text{per second}\end{pmatrix} = \begin{pmatrix}\text{change in momentum}\\ \text{in one collision}\end{pmatrix} \times \begin{pmatrix}\text{number of collisions}\\ \text{per second}\end{pmatrix}$$

$$-mv_x^2/L = (-2mv_x) \times (v_x/2L)$$



But by Newton's second law the change in momentum per second equals the average force. Therefore, $-mv_x^2/L$ = average force acting on molecule due to the wall; and by Newton's third law, $+mv_x^2/L$ = average force acting on the wall due to the molecule. Thus the pressure due to one molecule moving with a speed v_x is

$$P = F/A = F/L^2 = mv_x^2/L^3 = mv_x^2/V.$$

There are N molecules in the container. They will not all move with the same speed, but we only need to know the average speed in order to find the pressure. More precisely, we need the average of the square of their speeds in the x-direction, and we call this quantity, $\overline{v_x^2}$. The pressure on the wall due to N molecules will be N times the pressure due to one molecule, or $P = Nm\overline{v_x^2}/V$. We can express the square of the average speed in terms of the velocity components as follows: $\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}$. Also, if the motion is random, then there is no preferred direction of motion and $\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$. These last two expressions can be combined to give

$$\overline{v^2} = 3\overline{v_x^2} \quad \text{or} \quad \overline{v_x^2} = \frac{1}{3}\,\overline{v^2}.$$

By substituting this expression into our pressure formula, we get

$$P = \frac{1}{3}\,N\,\overline{mv^2}/\,V\,.$$

Recall that density is $D = \frac{\text{mass}}{\text{volume}} = \frac{Nm}{V}$

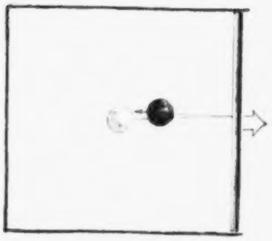
Our final expression for the pressure in terms of molecular speed v and density D is therefore

$$P = \frac{1}{3}\,D\overline{v^2}. \qquad (11.3)$$

This formula agrees with Boyle's law—pressure is proportional to density—if we can assume that the other factor on the right-hand side of Eq. (11.3), namely $\overline{v^2}$, is constant. Since the mass of the particles is constant anyway, this amounts to the same thing as assuming that the total kinetic energy of all the particles $(\frac{1}{2}Nm\overline{v^2})$ is constant.

Why should the total kinetic energy of a gas of particles remain constant when its pressure and density change? If we can answer that question, we will be able to say that the kinetic-theory model explains one of the properties of real gases, Boyle's law.

At first sight it would seem that the kinetic energy of the system would not remain constant when we change the pressure or density. Suppose we reduce the outside force that holds the piston in place. What will happen? The force on the piston resulting from the collisions of the particles will now be greater than the outside force, and the piston will start to move to the right.



As long as the piston was stationary, the particles did not do any work on it, and the piston did not do any work on the particles. But if the piston moves in the same direction as the force exerted on it by the particles, then the particles must be doing work on the piston. The energy needed to do this work must come from somewhere. But the only source of energy in our model is the kinetic energy of the particles. Therefore the kinetic energy of the particles must decrease.

Another way to look at this problem is to apply the laws of conservation of momentum and mechanical energy to the collisions between the particles and the piston. According to the principles stated in Chapter 10, if a light particle collides with a heavy piston which is moving in the same direction (that is, away from the particle), then the speed of the particle will be smaller when it bounces off, even if the collision is perfectly elastic. Therefore its kinetic energy is smaller.

If we had increased the outside force of the piston instead of decreasing it, just the opposite would happen. The piston would move to the left, doing work on the particles and increasing their kinetic energy. This result could also be predicted from the principles of conservation of momentum and mechanical energy.

The kinetic energy of the particles in our model is not constant when pressure and density change. On the contrary, it will increase when the pressure increases, and decrease when the pressure decreases. Therefore the model does not explain Boyle's law—unless we can find some reason why the kinetic energy of the particles should remain constant when the pressure changes.

In order to keep the kinetic energy of the particles constant, even though changes in pressure would tend to change the kinetic energy, we must provide some supply of energy. This energy supply must have the following two properties:

(1) it must add kinetic energy to the gas particles whenever they lose energy because the pressure decreases;

(2) it must take away kinetic energy whenever the particles increase their kinetic energy because the pressure increases.

In other words, the energy-supply must always act in such a way as to maintain a constant amount of kinetic energy in a gas.

If such an energy supply were included in the kinetic-theory model, this model would then provide a satisfactory explanation for the pressure of air and other gases.

The effect of temperature on gas pressure. Robert Boyle, writing in 1660 about the pressure of air, recognized that heating a gas would increase its volume. Many experiments were done throughout the eighteenth century on the expansion of gases by heat, but the results were not consistent enough to establish a quantitative relation between volume and temperature.

It was not until about 1800 that the law of thermal expansion of gases was definitely established by the French chemist Joseph-Louis Gay-Lussac (1778-1850). Gay-Lussac found that all the gases he studied—air, oxygen, hydrogen, nitro-

Maxwell suggested the following analogy: "When air is compressed the sides of the vessel are moving to meet the molecules like a cricket bat swung forward to meet the ball, and the molecules like the cricket ball rebound with a greater velocity than before. When air is allowed to expand the sides of the vessel are retreating like the cricketer's hands when he is stopping the ball and the molecules rebound with diminished velocity. Hence air becomes warmer when compressed and cooler when allowed to expand." (from notes for an unpublished lecture, in the Maxwell collection at Cambridge University)

See Study Guide 11.14

gen, nitrous oxide, ammonia, hydrochloric acid, sulfur dioxide and carbon dioxide—changed their volume by the same amount when the temperature changed by the same amount. The amount of volume change was always proportional to the change in temperature, as long as the pressure remained constant. On the other hand, if the volume (or density) was held constant, the change in pressure would always be proportional to the change in temperature.

The experimental data obtained by Power, Towneley, Boyle, Gay-Lussac and many other scientists can be summarized by a single equation, known as the ideal gas law

$$P = kD(t + 273) \tag{11.4}$$

where t is the temperature on the Celsius scale and k is a number which is constant when the pressure, volume and temperature of the same sample of gas are changed.

This equation is called the ideal gas law because it does not apply accurately to real gases except at very low pressures. It is not a law of physics in the same sense as the law of conservation of momentum, but rather a first approximation to the properties of real gases. It is not valid when the pressure is so high, or the temperature is so low, that the gas may change to a liquid.

On the Celsius scale, water freezes at 0° and boils at 100°, when the pressure is equal to normal atmospheric pressure. On the Fahrenheit scale, water freezes at 32° and boils at 212°.

Why does the number 273 appear in the ideal gas law? Simply because we are measuring temperature on the Celsius scale. If we had chosen to use the Fahrenheit scale, the equation for the ideal gas law would be

$$P = kD(t + 460)$$

where t is the temperature on the Fahrenheit scale. In other words, the fact that the number is 273 or 460 has no real significance, but depends on our choice of a particular scale for measuring temperature. However, it is significant that the pressure and volume of a gas depend on temperature in such a way that the product of pressure times volume would be zero when the temperature decreases to a certain value.

The value of this lowest possible temperature is -273.16° Celsius (459.69° Fahrenheit). Both experiment and thermodynamic theory have shown that it is impossible actually to cool anything all the way down to this temperature. However, temperature can be lowered in a series of cooling operations to within a small fraction of a degree above the limit.

Lord Kelvin proposed to define a new temperature scale, called the absolute temperature scale. Sometimes it is called the Kelvin scale. The absolute temperature T is equal to 273 degrees more than the Celsius temperature, t:

$$T = t + 273 \qquad (11.5)$$

The temperature $t = -273^{\circ}C$, which is unattainable, is now $T = 0$ on the absolute scale, and is called the absolute zero of temperature.

The ideal gas law may now be written in the simpler form:

$$P = kDT \qquad (11.6)$$

Note that the ideal gas law includes Boyle's law as a special case: when the temperature is held constant, the pressure is proportional to the density.

Heat and molecular kinetic energy. Now that we have redefined Boyle's law by adding the condition that the temperature must be kept constant when pressure and density change, we can go back to the kinetic-theory model and see what is needed to put it into agreement with the properties of gases. To do this, we compare the two equations

$$P = \frac{1}{3} D\overline{v^2} \text{ (theory)} \qquad (11.3)$$

and

$$P = kDT \text{ (experiment)}. \qquad (11.6)$$

These two equations are consistent only if we assume that

$$\frac{1}{3} D\overline{v^2} = kDT:$$

that is, that $\overline{v^2} = 3kT$.

If we multiply both sides of the last equation by $\frac{1}{2}$ m, we get the interesting result that

$$\frac{1}{2} m\overline{v^2} = \frac{3}{2} mkT.$$

Thus the theory implies that the average kinetic energy per particle is proportional to the absolute temperature!

We pointed out earlier that the kinetic-theory model of a gas would not be able to provide an explanation of Boyle's law unless there were some kind of energy supply which could keep the kinetic energy of the particles constant when the pressure changes. Now we know how this energy supply should be controlled: by a thermostat. If we simply keep the surroundings of the gas at a fixed temperature, then the average kinetic energy of the molecules will also remain fixed. Whenever the kinetic energy momentarily decreases (for example, during expansion) the temperature of the gas will drop below that of its surroundings. Heat will then flow into the gas until its temperature comes back up to the temperature of the surroundings. Whenever the kinetic energy momentarily increases (for example, during compression) the temperature of the gas will rise above that of its surroundings. Heat will then flow out of the gas until its temperature comes back down to the temperature of the surroundings. The natural tendency of heat to flow from hot bodies to cold bodies is just what we need to keep the average kinetic energy of the particles constant.

Q10 The pressure of a gas depends upon the square of the speed of the molecules because

a) the momentum changes upon impact with the wall.
b) the time between impacts is shorter for high speed molecules.
c) a combination of (a) and (b).
d) the area of the wall is measured in squared units of length.

Q11 If a piston is pushed into a container of gas, what will happen to the total kinetic energy of the molecules of gas?

Q12 Which of the following results when the ideal gas law is combined with the kinetic theory?

a) P is proportional to *T.*
b) P *is* proportional to v^2.
c) $\frac{1}{2}mv^2$ is proportional to T.

11.6 *The second law of thermodynamics and the dissipation of energy*. At the beginning of this unit we mentioned a basic philosophical theme of the Newtonian cosmology: the idea that the world is like a machine whose parts never wear out, and which never runs down. This idea inspired the search for conservation laws for matter and motion, and up to now it might seem that the search has been successful. We can measure "matter" by mass, and "motion" by momentum or kinetic energy. By 1850 the law of conservation of mass had been firmly established in chemistry, and the laws of conservation of momentum and energy had been firmly established in physics.

Our life runs down in sending
up the clock.
The brook runs down in sending
up our life.
The sun runs down in sending up
the brook.
And there is something sending
up the sun.
It is this backward motion toward the source,
Against the stream, that most we
see ourselves in,
The tribute of the current to
the source.
It is from this in nature we
are from.
It is most us.
(Robert Frost, *West-Running Brook* p. 37)

Yet these conservation laws could not banish the suspicion that somehow the world *is* running down, and the parts of the machine *are* wearing out. Energy may be conserved in burning fuel, but it loses its usefulness as the heat diffuses into the atmosphere. Mass may be conserved in scrambling an egg, but structure is lost. In these transformations, something is conserved, but also something is lost. Some processes are *irreversible*—they will not run backwards.

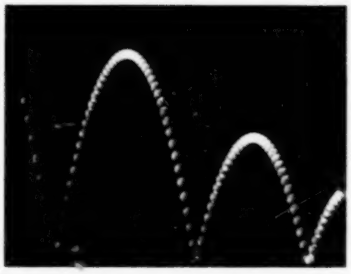
The first attempts to formulate quantitative laws for irreversible processes in physics were stimulated by the development of steam engines. During the eighteenth and nineteenth centuries, the efficiency of steam engines—the amount of mechanical work that could be obtained from a given amount of fuel energy—was steadily increased (see Sec. 10.6). In 1824, a young French engineer, Sadi Carnot, published a short book entitled Reflections on the Motive Power of Fire. Carnot raised the question: what is the maximum efficiency of an engine? By careful analysis of the flow of heat in the engine, Carnot proved that there is a maximum efficiency, always less than 100%. There is a fixed upper limit on the amount of mechanical energy that can be obtained from a given amount of heat by using an engine, and this limit can never be exceeded regardless of what substance—steam, air, or anything else—is used in the engine.

Even more ominous than the existence of this limit on efficiency was Carnot's conclusion that all real engines fail to attain the theoretical limit in practice. The reason is that whenever a difference of temperature exists between two bodies, or two parts of the same body, there is a possibility of doing work by allowing heat to expand a gas as the heat flows from one body to the other. But if heat flows by itself from a hot body to a cold body, and we do not design our engine properly, we will lose the chance of doing work that might have been done.

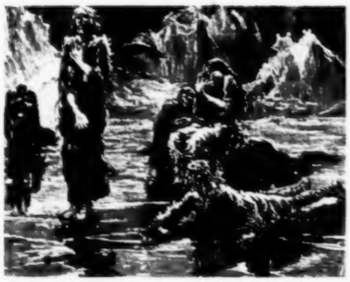
Carnot's analysis of steam engines shows that the process of equalization of temperature by the flow of heat from hot bodies to cold bodies represents a waste of mechanical energy. This is what we mean when we say that energy is "degraded" or "dissipated"—the total amount of energy is always the same, but energy tends to transform itself into less useful forms.

After the discovery of the law of conservation of energy, Carnot's conclusions about steam engines were incorporated into the new theory of heat (thermodynamics) and became known as the second law of thermodynamics. This law has been stated in various ways, all of which are roughly equivalent, and express the idea that the tendency of heat to flow from hot to cold makes it impossible to obtain the maximum amount of mechanical energy from a given amount of heat.

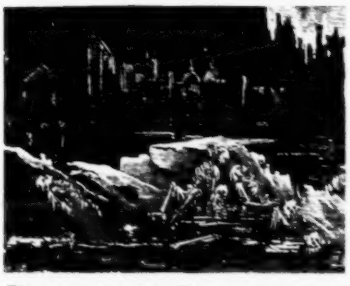
Carnot's analysis of steam engines implies more than this purely negative statement, however. In 1852, Lord Kelvin generalized the second law of thermodynamics by asserting that there is a universal tendency in nature toward the dissipation of energy. Another way of stating this principle was suggested by Rudolf Clausius, in 1865. Clausius introduced a new concept, entropy, which he defined in terms of the heat transferred from one body to another. We will not discuss the technical meaning of entropy in thermodynamics but simply state that whenever heat flows from a hot body to a cold body, entropy increases. It also increases whenever mechanical energy is changed into internal energy, as in inelastic collisions and frictional processes. All these changes can be identified with increasing disorder of the system. And indeed entropy can be defined as a measure of the disorder of a system. So the generalized version of the second law of thermodynamics, as stated by Clausius, is simply: the entropy of a system always tends to increase.



Irreversible processes are processes for which entropy increases; hence they cannot be run backwards without violating the second law of thermodynamics. For example, heat will not flow by itself from cold bodies to hot bodies; a ball dropped on the floor will not bounce back higher than its original position; and an egg will not unscramble itself. All these (and many other) events, which could take place without violating any of the principles of Newtonian mechanics, are forbidden by the second law of thermodynamics.



"La misérable race humaine périra par le froid."



"Ce sera la fin."

Lord Kelvin predicted, on the basis of his principle of dissipation of energy, that all bodies in the universe would eventually reach the same temperature by exchanging heat with each other. When this happens, it will be impossible to produce any useful work from heat, since work can only be done when heat flows from a hot body to a cold body. The sun and other stars would eventually grow cold, all life on earth would cease and the universe would be dead. This "heat death," which seemed to be an inevitable consequence of thermodynamics, aroused some popular interest at the end of the nineteenth century, and was described in several books written at that time, such as H. G. Wells' The Time Machine. The American historian Henry Adams, who learned about thermodynamics through the works of one of America's greatest scientists, J. Willard Gibbs, argued that the second law could be applied to human history. He wrote a series of essays which were published under the title The Degradation of the Democratic Dogma. The French astronomer Camille Flammarion wrote a book describing all the possible ways in which the world could end; we have reproduced from his book two illustrations showing an artist's conception of the heat death.

Q13 The "heat death of the universe" refers to a state

a) in which all mechanical energy has been transformed into heat energy.
b) in which all heat energy has been transformed into other forms of energy.
c) in which the temperature of the universe decreases to absolute zero.
d) in which the supply of coal and oil has been used up.

Q14 Which of the following statements are consistent with the second law of thermodynamics?

a) Heat does not naturally flow from cold bodies to hot bodies.
b) Energy tends to transform itself into less useful forms.
c) No engine can transform all its heat input into mechanical energy.
d) Most processes in nature are reversible.

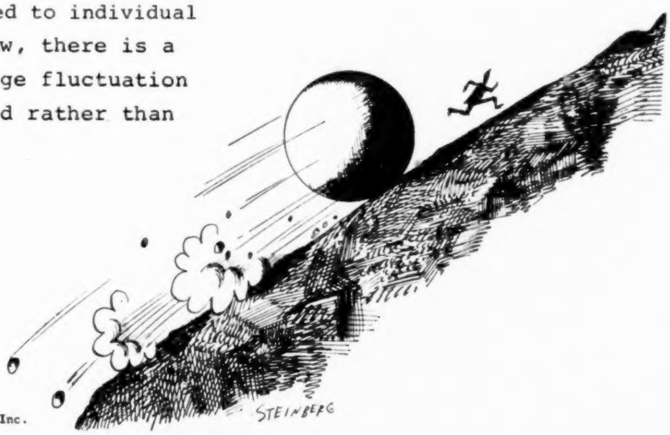
11.7 Maxwell's demon and the statistical view of the second law of thermodynamics. Is there any way of avoiding the heat death? Is irreversibility a basic law of physics, or only an approximation based on our limited experience of natural processes?

See "Maxwell's Demon" in Project Physics Reader 3

The Austrian physicist Ludwig Boltzmann used the kinetic theory of gases to investigate the nature of irreversibility, and concluded that the tendency toward dissipation of energy is not an absolute law of physics but only a statistical one. Boltzmann argued that if one were to list all the possible arrangements of molecules in a gas, nearly all of them would have to be considered "disordered." Only a few of them, for example, would have all the molecules in one corner of an otherwise-empty container. It is to be expected that if we start from an ordered arrangement of molecules, the arrangement will inevitably become less ordered, simply because most possible arrangements are random. Similarly, if we put a hot body (whose molecules are moving rapidly) in contact with a cold body (whose molecules are moving slowly), it is almost certain that after a short time both bodies will have nearly the same temperature, simply because there are many more possible arrangements of molecular speeds in which fast and slow molecules are mixed together, than arrangements in which most of the fast molecules are in one place and most of the slow molecules are in another place.

To illustrate Boltzmann's argument we might consider shuffling a pack of cards. Most possible arrangements of the cards are more or less disordered. If we start with an ordered arrangement—for example, the cards sorted by rank and suit—then shuffling would almost certainly lead to a more disordered arrangement. Nevertheless it does occasionally happen that a player is dealt 13 spades even if no one has stacked the deck.

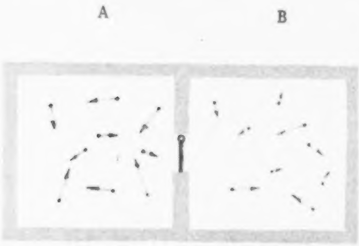
According to Boltzmann's view, it is almost certain that disorder will increase in any natural process that we can actually observe. The second law is therefore a statistical law that applies to collections of large numbers of molecules, but may have no meaning when applied to individual molecules. Since it is a statistical law, there is a remote possibility that a noticeably large fluctuation may occur in which energy is concentrated rather than dissipated.



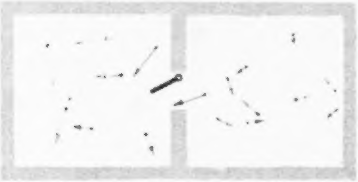
Drawing by Steinberg; © 1963, The New Yorker Magazine, Inc.

For example, the molecules in a glass of water are usually moving randomly in all directions but they might all just happen to move in the same direction at the same time. The water would then jump out of the glass. (The glass would have to move downward at the same time since momentum must still be conserved.) In this case a disordered motion has suddenly turned into an ordered motion; entropy has decreased instead of increased, and the second law of thermodynamics (regarded as an *absolute* law of physics) has been violated. Such large fluctuations seem extremely unlikely, yet if they can occur at all we must recognize that the second law is not a fundamental law of physics.

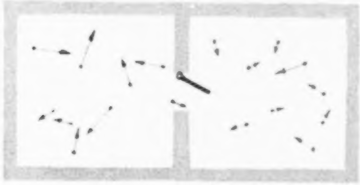
How Maxwell's "demon" could use a small, massless door to *increase* the order of a system and make heat flow from a cold gas to a hot gas.



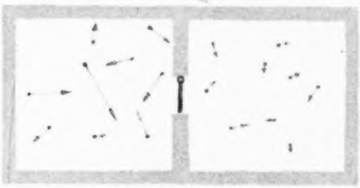
(a) **Initially the average KE of molecules is greater in A.**



(b) **Only fast molecules are allowed to go from B to A.**



(c) **Only slow molecules are allowed to go from A to B.**



(d) **As this continues, the average KE in A increases and the average KE in B decreases.**

Maxwell proposed a "thought experiment" to show how the second law of thermodynamics could be violated by an imaginary person who could observe individual molecules and sort them out, thereby causing heat to flow from cold to hot. Suppose a container of gas is divided into two parts, A and B, by a diaphragm. Initially the gas in A is hotter than the gas in B. This means that the molecules in A have greater *average* speeds than those in B. However, since the speeds are distributed according to Maxwell's distribution law (Sec. 11.3), a few molecules in A will have speeds less than the average in B, and a few molecules in B will have speeds greater than the average in A.

Maxwell saw that there would be a possibility of making heat flow from a cold gas to a hot gas because of this overlapping of the distributions for gases at different temperature. "Now conceive a finite being," Maxwell suggested, "who knows the paths and velocities of all the molecules by simple inspection but who can do no work except open and close a hole in the diaphragm by means of a slide without mass." (If the slide has no mass, no work will be needed to move it.) Let this "finite being" observe the molecules in A, and when he sees one coming whose speed is less than the average speed of the molecules in B, let him open the hole and let it go into B. Now the average speed of the molecules of B will be even lower than it was before. Next, let him watch for a molecule of B, whose speed is greater than the average speed in A, and when it comes to the hole let him draw the slide and let it go into A. Now the average speed in A will be even higher than it was before. Maxwell concludes:

> Then the number of molecules in A and B are the same as at first, but the energy in A is increased and that in B diminished, that is, the hot system has got hotter and the cold colder and yet no work has been done, only the intelligence of a very observant and neat-fingered being has been employed.

The imaginary "being who knows the paths and velocities of all the molecules" has come to be known as "Maxwell's Demon." Maxwell's thought experiment shows that if there were any way to sort out individual molecules, the principle of dissipation of energy could be violated. Some biologists have suggested that certain large molecules, such as enzymes, may actually be able to guide the motions of smaller molecules, building up ordered molecular systems in living beings in just this way.

Q15 In each of the following, which situation is more ordered?

a) an unbroken egg; a scrambled egg
b) a glass of ice and warm water; a glass of water at uniform temperature

Q16 True or false?

a) Maxwell's demon was able to circumvent the second law of thermodynamics.
b) Modern physics has made a Maxwell's demon.
c) Maxwell believed in the existence of his demon.

11.8 Time's arrow and the recurrence paradox. Does time have a direction? Is there any real difference between going forward and going backward in time? These questions lie at the root of the problem of irreversibility.

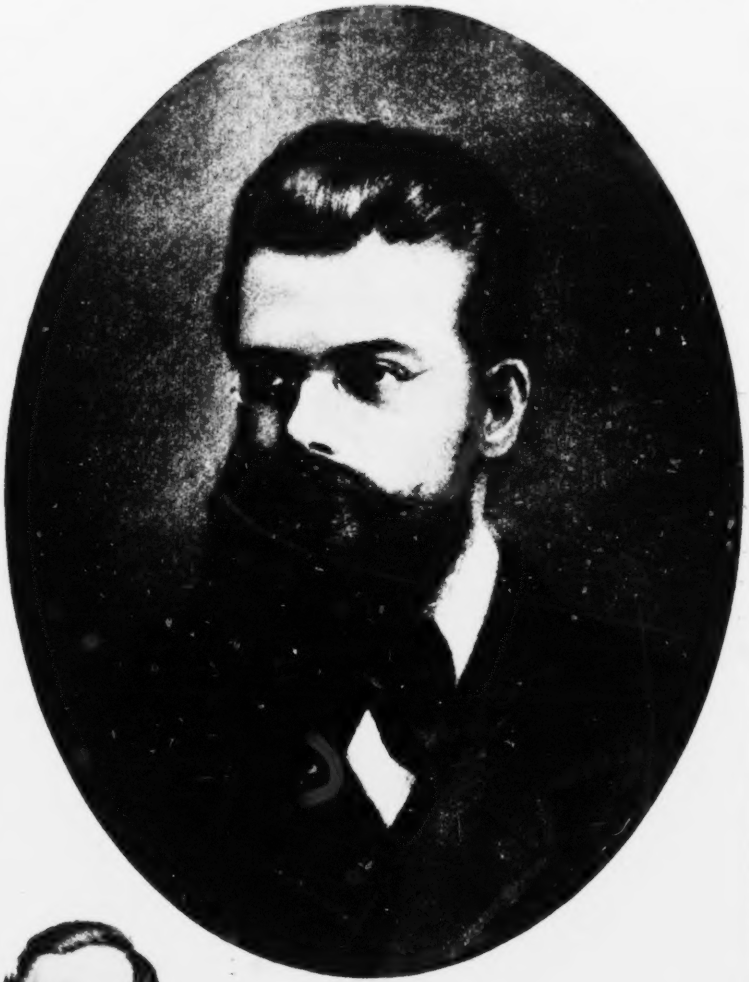
See "The Arrow of Time" in Project Physics Reader 3

Toward the end of the nineteenth century, a small but influential group of scientists began to question the basic philosophical assumptions of Newtonian mechanics and even the very idea of atoms. The Austrian physicist Ernst Mach criticized Newton's concepts of force, mass and absolute space, and argued that scientific theories should not depend on assuming the existence of things (such as atoms) which could not be directly observed. Typical of the attacks on atomic theory is the argument which the mathematician Ernst Zermelo and others advanced against kinetic theory: the second law of thermodynamics is an absolutely valid law of physics because it agrees with all the experimental data, but kinetic theory allows the possibility of exceptions to this law; hence kinetic theory must be wrong.

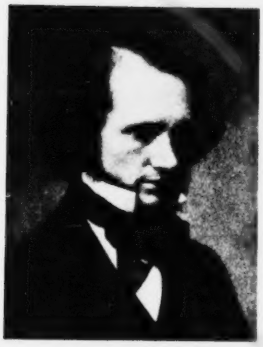
The critics of kinetic theory could point to two apparent contradictions between the kinetic theory (or in fact any molecular theory based on Newton's laws of mechanics) and the principle of dissipation of energy: the reversibility paradox and the recurrence paradox. Both paradoxes are based on possible exceptions to the second law. In considering their relevance to the validity of kinetic theory, we have to decide whether it is good enough to show that these exceptions would occur extremely rarely, or whether we must exclude them entirely.

A few of the scientists who made significant contributions to the development of the kinetic theory of gases and thermodynamics

1



2



3

4

5

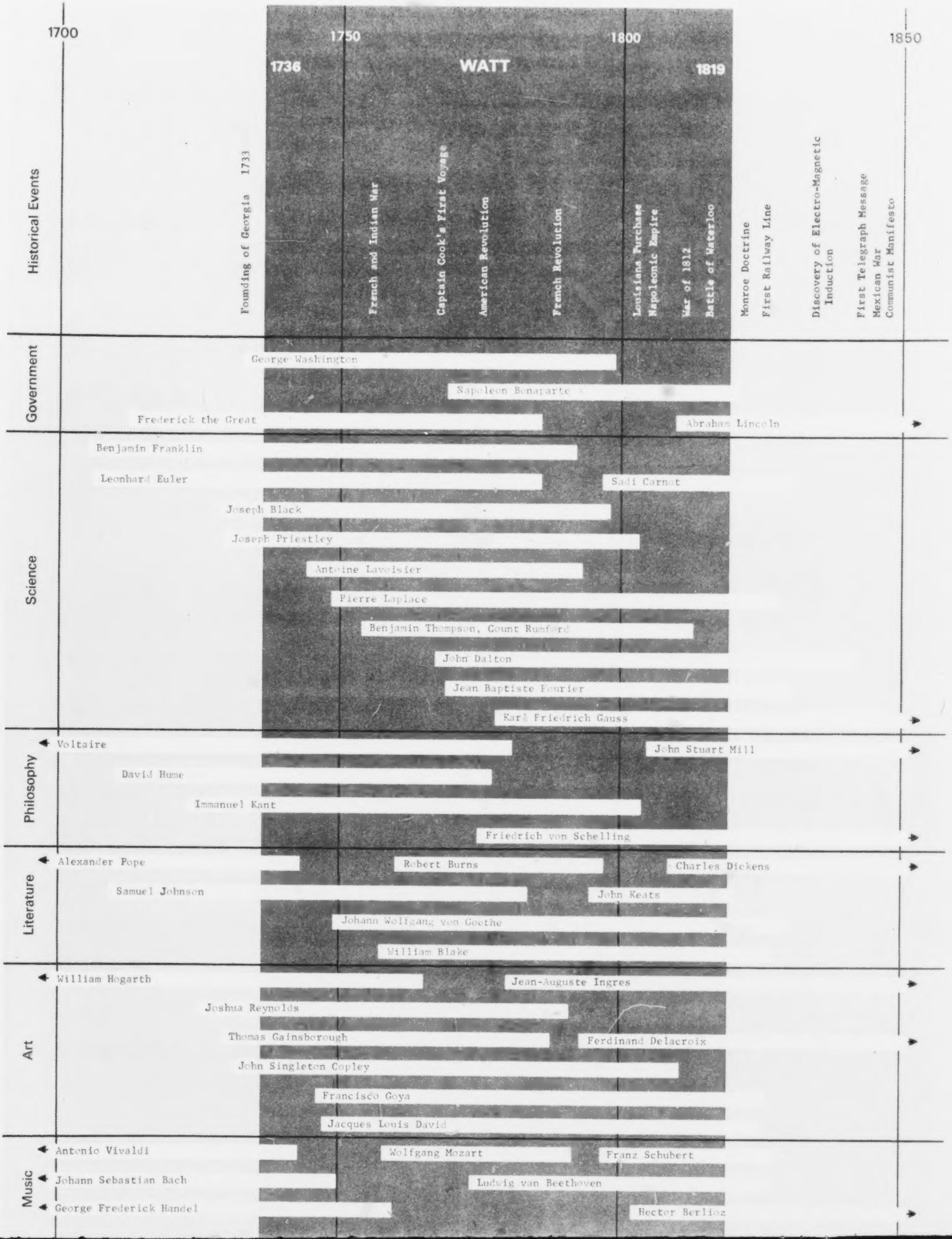
1 James Clerk Maxwell

2 Ludwig Boltzmann

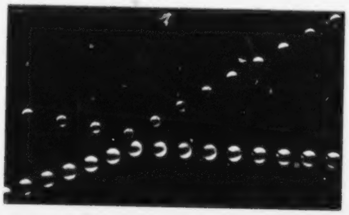
3 William Thomson (Lord Kelvin)

4 Sadi Carnot

5 Rudolf Clausius

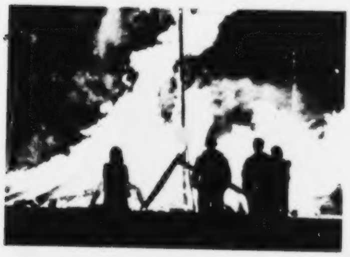

1700
1750
1800
1850
1736
WATT
1819
Historical Events
Founding of Georgia 1733
French and Indian War
Captain Cook's First Voyage
American Revolution
French Revolution
Louisiana Purchase
Napoleonic Empire
War of 1812
Battle of Waterloo
Monroe Doctrine
First Railway Line
Discovery of Electro-Magnetic Induction
First Telegraph Message
Mexican War
Communist Manifesto
Government
George Washington
Napoleon Bonaparte
Frederick the Great
Abraham Lincoln
Science
Benjamin Franklin
Leonhard Euler
Sadi Carnot
Joseph Black
Joseph Priestley
Antoine Lavoisier
Pierre Laplace
Benjamin Thompson, Count Rumford
John Dalton
Jean Baptiste Fourier
Karl Friedrich Gauss
Philosophy
Voltaire
John Stuart Mill
David Hume
Immanuel Kant
Friedrich von Schelling
Literature
Alexander Pope
Robert Burns
Charles Dickens
Samuel Johnson
John Keats
Johann Wolfgang von Goethe
William Blake
Art
William Hogarth
Jean-Auguste Ingres
Joshua Reynolds
Thomas Gainsborough
Ferdinand Delacroix
John Singleton Copley
Francisco Goya
Jacques Louis David
Music
Antonio Vivaldi
Wolfgang Mozart
Franz Schubert
Johann Sebastian Bach
Ludwig van Beethoven
George Frederick Handel
Hector Berlioz

11.8


The reversibility paradox: Can a model based on reversible events explain a world in which so many events are irreversible?

The reversibility paradox was discovered in the 1870's by Lord Kelvin and Josef Loschmidt, both of whom were supporters of atomic theory; it was not regarded as a serious objection to the kinetic theory until the 1890's. The paradox is based on the simple fact that Newton's laws of motion are reversible in time. Imagine that we could take a motion picture of the molecules of a gas, colliding elastically according to the assumptions of kinetic theory. When we showed the motion picture, there would be no way to tell whether it was being run forward or backward—either way would show valid sequences of collisions. But motion pictures of interactions involving large objects (containing many molecules) do have obvious differences between forward and backward time directions.

There is nothing in Newton's laws of motion which distinguishes going backward from going forward in time. How can the kinetic theory explain irreversible processes if it is based on laws of motion which are reversible? The existence of irreversible processes seems to indicate that time flows in a definite direction—from past to future—in contradiction to Newton's laws of motion.



As Lord Kelvin expressed the paradox,

> If...the motion of every particle of matter in the universe were precisely reversed at any instant, the course of nature would be simply reversed for ever after. The bursting bubble of foam at the foot of a waterfall would reunite and descend into the water; the thermal motions would reconcentrate their energy, and throw the mass up the fall in drops reforming into a close column of ascending water. Heat which had been generated by the friction of solids and dissipated by conduction, and radiation with absorption, would come again to the place of contact, and throw the moving body back against the force to which it had previously yielded. Boulders would recover from the mud the materials required to rebuild them into their previous jagged forms, and would become reunited to the mountain peak from which they had formerly broken away. And if also the materialistic hypothesis of life were true, living creatures would grow backwards, with conscious knowledge of the future, but no memory of the past, and would become again unborn. But the real phenomena of life infinitely transcend human science; and speculation regarding consequences of their imagined reversal is utterly unprofitable.

Kelvin himself, and later Boltzmann, used statistical ideas to explain why we do not observe such reversals. There are a large number of possible disordered arrangements of water molecules at the bottom of a waterfall; only an extremely small number of these arrangements would lead to the process described in the above quotation if we could reverse the velocity of every molecule. Reversals of this kind are possible in principle, but very unlikely.

The recurrence paradox revived an idea that had been frequently used in ancient philosophies: the myth of the "eternal return." According to this myth, the history of the world is cyclic; all historical events are repeated over and over again, and the people who are now dead will someday be born again and go through the same life. The German philosopher Friedrich Nietzsche was convinced of the truth of this idea, and even tried to prove it using the principle of conservation of energy. He wrote:

> If the universe may be conceived as a definite quantity of energy, as a definite number of centres of energy—and every other concept remains indefinite and therefore useless—it follows therefrom that the universe must go through a calculable number of combinations in the great game of chance which constitutes its existence. In infinity, at some moment or other, every possible combination must once have been realized; not only this, but it must have been realized an infinite number of times.

Nietzsche claimed that his proof of the eternal return refuted the theory of the heat death. At about the same time, in 1889, the French mathematician Henri Poincaré published a similar theorem on the recurrence of mechanical systems. According to Poincaré, his recurrence theorem implied that while the universe might undergo a heat death, it would ultimately come alive again:

> A bounded world, governed only by the laws of mechanics, will always pass through a state very close to its initial state. On the other hand, according to accepted experimental laws (if one attributes absolute validity to them, and if one is willing to press their consequences to the extreme), the universe tends toward a certain final state, from which it will never depart. In this final state, which will be a kind of death, all bodies will be at rest at the same temperature.
>
> ...the kinetic theories can extricate themselves from this contradiction. The world, according to them, tends at first toward a state where it remains for a long time without apparent change; and this is consistent with experience; but it does not remain that way forever; ...it merely stays there for an enormously long time, a time which is longer the more numerous are the molecules. This state will not be the final death of the universe, but a sort of slumber, from which it will awake after millions of centuries.
>
> According to this theory, to see heat pass from a cold body to a warm one, it will not be necessary to have the acute vision, the intelligence, and the dexterity of Maxwell's demon; it will suffice to have a little patience.

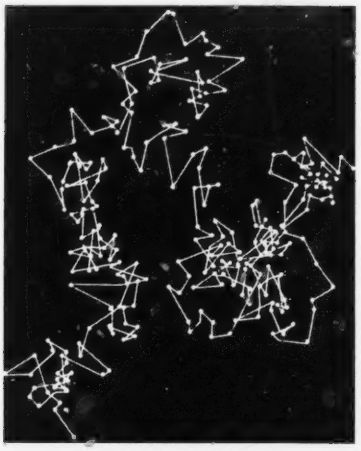
Though Poincaré was willing to accept the possibility of a violation of the second law after a very long time, others were less tolerant. In 1896, Ernst Zermelo (at that time a student of Max Planck) published a paper attacking not only

The World's great age begins
anew,
The golden years return,
The earth doth like a snake
renew
His winter weeds outworn...
Another Athens shall arise
And to remoter time
Bequeath, like sunset to the
skies,
The splendour of its prime...

Percy Bysshe Shelley, "Hellas" (1822)

the kinetic theory but the mechanistic conception of the world in general, on the grounds that it contradicted the second law. Boltzmann replied, repeating his earlier explanations of the statistical nature of irreversibility. When these did not satisfy Zermelo, Boltzmann (half-seriously) proposed the following hypothesis: the history of the universe is really cyclic, so that the energy of all its molecules must eventually be reconcentrated in order to start the next cycle. During this process of reconcentration, all natural processes will go backward, as described by Kelvin (above). However, the human sense of time depends on natural processes going on in our own brains. If the processes are reversed, our sense of time will also be reversed. Therefore we could never actually observe "time going backward" since we would be going backward along with time.

The final outcome of the dispute between Boltzmann and his critics was that both sides were partly right and partly wrong. Mach and Zermelo were correct in their belief that molecular and atomic processes cannot be adequately described by Newton's laws of mechanics; we will come back to this subject in Unit 5. Gases are not collections of little billiard balls. But Boltzmann was right in his belief in the usefulness of the molecular model; the kinetic theory is very nearly correct except for those properties involving the detailed structure of molecules.



Record of a particle in Brownian motion. Successive positions, recorded every 20 seconds, are connected by straight lines. The actual paths between recorded positions would be as erratic as the overall path.

In 1905, Albert Einstein pointed out that the fluctuations predicted by kinetic theory should produce an effect which could be observed and measured quantitatively: the motion of very small particles in liquids. Subsequent studies of this motion (called "Brownian motion" after Robert Brown, the English botanist who had observed it in 1828) confirmed Einstein's theoretical calculations. This new success of kinetic theory, along with the discoveries in radioactivity and atomic physics at the beginning of the twentieth century, persuaded almost all the skeptics that atoms really exist. But the problem of irreversibility and the question of whether the laws of physics must distinguish between past and future, are issues that still interest physicists today.

Q17 The kinetic energy of a falling stone is transformed into heat when the stone strikes the ground. Obviously this is an irreversible process; we never see the heat transform into kinetic energy of the stone, so that the stone rises off the ground. The reason that the process is irreversible is that

a) Newton's laws of motion prohibit the reversed process.
b) there is a very small probability that the disordered molecules will happen to arrange themselves in the way necessary for the reversed process to occur.
c) the reversed process would not conserve energy.

Q18 Which of the following is a reversible process?

a) a pendulum swinging in air.
b) water falling in a cataract.
c) two molecules colliding perfectly elastically.
d) an ice cube melting in a glass of warm water.

Q19 The recurrence theorem states that any given arrangement of molecules will be repeated after a long enough time. The theorem

a) appears to contradict the principle of dissipation of energy.
b) was discovered by ancient philosophers.
c) was disproved by Poincaré.
d) applies only to the molecules of living systems.

11.1 List some of the directly observable properties of gases.

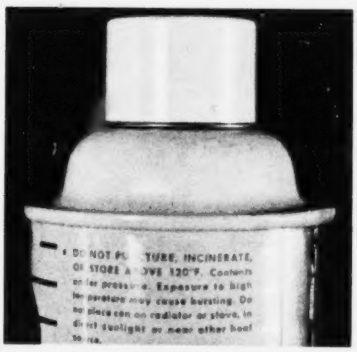
11.2 What could kinetic theorists explain about gases?

11.3 Where did Newtonian mechanics run into difficulties in explaining the behavior of molecules?

11.4 Distinguish between two uses of the word "model" in science.

11.5 Randomness can be used in predicting the results of flipping a large number of coins. Give some other examples where randomness is useful.

11.6 The speed of sound in a gas is about the same as the average speed of the gas molecules. Is this a coincidence? Discuss.


DO NOT PU TURE, INCINERATE,
OR STORE A OVE 120°F. Contents
under pressure. Exposure to high
temperature may cause bursting. Do
not place can on radiator or stove, in
direct sunlight or near other heat
sources.

11.7 Consider the curves showing Maxwell's distribution of molecular speeds.

a) All show a peak.
b) The peaks move toward higher speed at higher temperatures.
c) They are not symmetrical like normal distribution curves.

Explain these characteristics on the basis of the kinetic model.

11.8 Many products are now sold in spray cans. Explain in terms of the kinetic theory of gases why it is dangerous to expose the cans to high temperatures.

11.9 Benjamin Franklin in 1765 observed that not more than a teaspoonful of oil covered half an acre of a pond. Suppose that one cubic centimeter of oil forms a continuous layer one molecule thick that just covers an area on water of 1000 square meters.

a) How thick is the layer?
b) What is the diameter of a single molecule of the oil?

11.10 How did Clausius modify the simple "model" for a gas? What was this new model able to explain?

11.11 How did Josef Loschmidt estimate the size of a molecule?

11.12 The atmospheric pressure of air is balanced by a column of mercury of height 0.76 meters or by 10.5 meters of water. Air is approximately a thousand times less dense than water. Why can you not say the atmosphere is only 10,000 meters deep?

11.13 The spike heel on a girl's shoe is a square, one centimeter on an edge. If her mass is 50 kilograms, how many atmospheres of pressure are exerted when she balances on one heel?

11.14 If a light particle rebounds from a massive, stationary piston with almost no loss of speed, then, according to the principle of Galilean relativity, it would still do so from a moving piston in the frame of reference of the moving piston. Show that the rebound speed as measured in the laboratory would be less from a retreating piston, as is claimed at the top of p. 95. (Hint: Express the speed of the particle relative to the piston in terms of their speeds in the laboratory frame.)

11.15 Clausius' statement of the second law of thermodynamics is: "Heat will not of its own accord pass from a cooler to a hotter body." Show in words how a refrigerator can operate.

11.16 When a gas is compressed by pushing in a piston, its temperature increases.

a) Explain this fact in two ways: first, by using the first law of thermodynamics and second, by using the kinetic theory of gases.

b) The compressed air eventually cools down to the same temperature as the surroundings. Explain this heat transfer in terms of molecular collisions.

11.17 Why, if there is a tendency for heat to flow from hot to cold, will not the universe eventually reach absolute zero?

11.18 How did Maxwell's demon hope to circumvent the second law of thermodynamics?

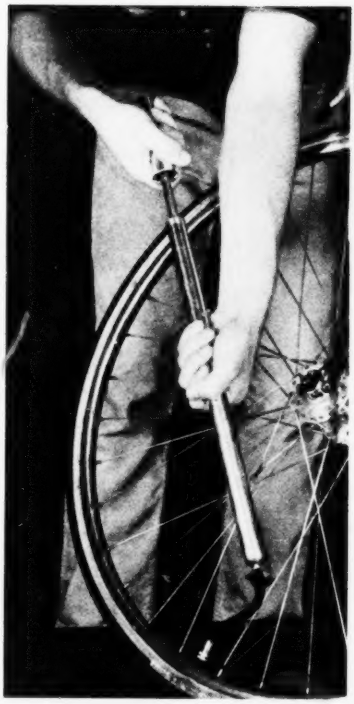
11.19

a) Explain what is meant by the statement that Newton's laws of motion are time-reversible.

b) Describe how a paradox arises when the time-reversibility of Newton's laws of motion is combined with the second law of thermodynamics.

11.20 Since molecular motions are random, one might expect that any given arrangement of molecules would recur if he waited long enough. Explain how a paradox arises when this prediction is combined with the second law of thermodynamics.

11.21 Many philosophical and religious systems of the Far East and the Middle East include the ideas of eternal return and resurrection. Read about some of these philosophies and discuss them in the light of your knowledge of the second law of thermodynamics.



Chapter 12 Waves

Section		Page
12.1	Introduction	113
12.2	Properties of waves	114
12.3	Wave propagation	117
12.4	Periodic waves	118
12.5	When waves meet	121
12.6	A Two-source interference pattern	124
12.7	Standing waves	128
12.8	Wave fronts and diffraction	131
12.9	Reflection	134
12.10	Refraction	138
12.11	Sound waves	141

12.1 Introduction. Waves are all around us. Water waves, especially, whether giant rollers in the middle of the ocean or gently formed rain ripples on a still pond, are sources of wonder or amusement. If the earth's crust shifts, violent waves cause tremors thousands of miles away. A musician plucks a guitar string and sound waves pulse against our ears. Someone stumbles on a crowded dance floor and a wave of bumping or crowding spreads through the adjacent dancers. Wave disturbances may come in a concentrated bundle like the shock front from a single clap of the hands or from an airplane flying at supersonic speeds. Or the disturbances may come in succession like the train of waves sent out from a steadily vibrating source, such as an alarm clock.

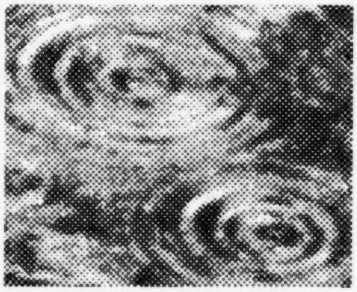
As physics has progressed over the last hundred years, vibrations and waves of a less obvious kind have been discovered. Electromagnetic waves in particular have been found to be fundamental to nearly everything we can sense about our universe. Most of our explanations of energy transfer involve waves.

So far we have been thinking of motion in terms of individual particles. As we begin to study the cooperative motion of collections of particles, we shall recognize how intimately related are the particle and wave models we make of events in nature.

If you look at a black and white photograph in a newspaper or magazine with a magnifying glass, you discover that the picture is made up of many little black dots printed on a white page (up to 20,000 dots per square inch). If you do not use the magnifier, you will not see the individual dots, but a pattern with all possible shadings between completely black and completely white. The two views emphasize different aspects of the same thing.

In much the same way, the physicist often has available several ways of viewing events. For the most part, a particle view has been emphasized in the first three units. In Unit 2, for example, we treated each planet as a particle experiencing the sun's gravitational attraction. The behavior of the solar system was described in terms of the positions, velocities and accelerations of point-like objects, but this viewpoint is far from a complete description of our planetary neighbors.

In the last chapter we saw two different descriptions of a gas. One was in terms of the behavior of the individual particles making up the gas. We used Newton's laws of motion to describe what each particle does, and then we used average



A small section from the lower right of the photograph on the opposite page.

values to describe the behavior of the gas as a whole. But we also discussed a gas with the aid of concepts such as pressure, temperature, heat and entropy, which refer to a sample of gas as a whole. This is the viewpoint of thermodynamics, which does not depend on assuming Newton's laws or even the existence of particles. Each of these viewpoints served a useful purpose and helped us to understand what we cannot directly see.

See "Introduction to Waves" and "What is a Wave" in Project Physics Reader 3

Now we are about to study waves, and once again we find alternative points of view. Most of the waves discussed in this chapter can be described in terms of the behavior of particles, but we also want to understand waves as disturbances traveling in a continuous medium. We want, in other words, to see the picture as a whole, not only individual dots.

12.2 Properties of waves. What is a wave? We begin our study of waves with a simple example. Suppose that two people are holding opposite ends of a rope. One person snaps the rope up and down quickly. That puts a sort of hump in the rope, which travels along the rope toward the other person. The traveling hump is a wave.

High-speed photograph of ripples produced by a milk drop.

Originally, the rope is held motionless. The height of each point on the rope depends only upon its position along the rope. When one person snaps the rope, he creates a rapid change in the height of one end, and the disturbance then moves away from its source. The height of each point on the rope depends upon time as well as position.

The disturbance is a pattern of displacement along the rope. The motion of the displacement pattern is an example of a wave. The snapping of the one end is the source of the wave, and the rope is the medium in which the wave moves. These four terms are common to all mechanical wave situations.

Consider another example. Drop a pebble into a pool of still water, and a series of circular crests and troughs spread over the water's surface. This moving displacement pattern of the water surface is a wave. The pebble is the source, the moving pattern of crests and troughs is the wave and the water surface is the wave's medium.

In rope waves and water waves, we can see the media, the source and the disturbance. However, in our analysis we want to concentrate on the wave—the moving pattern. Each of these waves consists of changing displacements from the equilibrium height of successive parts of the medium. Thus we can refer to these as waves of displacement.

If we can see the medium and recognize the displacements, then we can see waves. As we proceed, we should be prepared to find waves in media we cannot see (such as air) or which are disturbances in properties we cannot detect with our eyes (such as pressure, or electric fields).

A loose spring coil can be used to demonstrate three differend kinds of waves. If the end of the spring is moved from side to side, as in Fig. 12.1 (a), a wave of side-to-side displacement will travel along the spring. If the end of the spring is moved back and forth, as in Fig. 12.1 (b), a wave of back-and-forth displacement will travel along the spring. If the end of the spring is twisted, a wave of angular displacement will travel along the spring. Waves like Fig. 12.1 (a), in which the displacements of the spring are perpendicular to the direction the wave travels, are called transverse waves. Waves like Fig. 12.1 (b), in which the displacements are in the direction the wave travels, are called longitudinal waves. And waves like Fig. 12.1 (c), in which the displacements are twisting in the direction of travel of the wave, are called torsional waves.

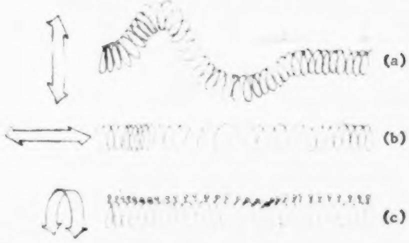


Fig. 12.1 Three types of waves on a coil spring. In (c), small markers have been put on the top of each coil.

All three can be set up in solids. In fluids, however, transverse and torsional waves die out very quickly if they can be produced at all. Thus sound waves in air and in water are longitudinal—the molecules of the medium are displaced back and forth along the direction that the sound travels.

It is often useful to make a graph of wave patterns. It is important to note that the graph always has a transverse appearance, even if it represents a longitudinal or torsional wave. Thus in Fig. 12.2 the pattern of compressions in a sound wave is represented by a graph. The graph line goes up and down to represent the increasing and decreasing density of the air, not to represent an up and down motion of the air.

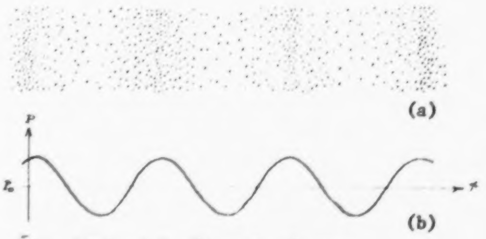


Fig. 12.2 (a) "Snapshot" representation of a sound wave progressing to the right. The dots represent the density of air molecules. (b) Graph of air pressure P vs. position x at the instant of the snapshot. P_o is the normal, undisturbed value of pressure.

A complete description of transverse waves involves a variable which descriptions of longitudinal or torsional waves do not: the direction of displacement. The displacements of a longitudinal wave can be in only one direction—the direction of travel of the wave. Similarly, the angular displacements of a torsional wave can be around only one axis—the direction of travel of the wave. But the displacements of a transverse wave can be in any and all of an infinite number of directions. This is easily seen on a rope by shaking one end around randomly instead of straight up and down or straight left and right. For simplicity, the diagrams of rope and spring waves in this chapter have shown

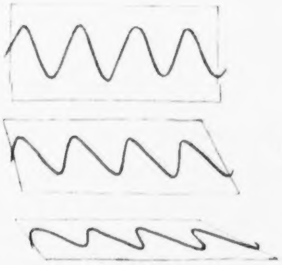


Fig. 12.3 Different polarization planes of a transverse wave.

transverse displacements consistently in a single plane.

When the displacement pattern of a transverse wave lies in a single plane, the wave is said to be polarized. For waves on ropes and springs we can observe the polarization directly. However, there is a general way of identifying a polarized wave, whether we can see the wave directly or not: find some effect of the wave which depends on angular position. An example of the principle is illustrated in the diagram below, where the interaction of a rope wave with a slotted board is shown to depend on the angle of the slotted board. Each of the three sketches begins with the same wave.

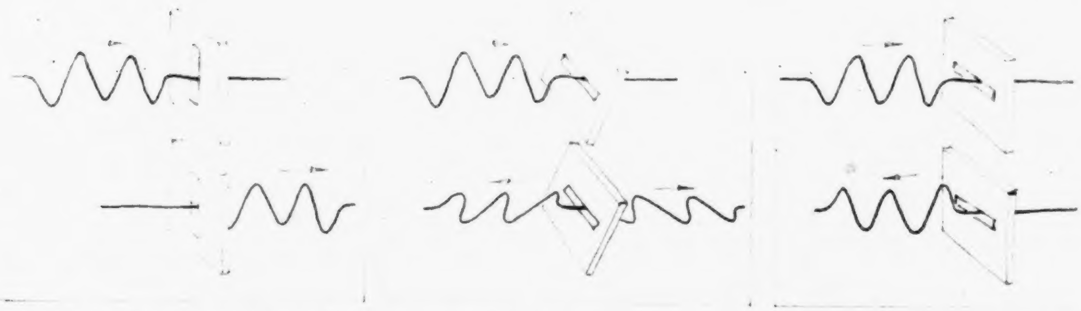


Fig. 12.4 The interaction of a polarized rope wave is sketched for three different orientations of a slotted board. The same short wave train approaches the slotted board in each of the three sketches. Depending on the orientation of the slot, the train of waves (a) goes entirely through the slot; (b) is partly reflected and partly transmitted; or (c) is completely reflected.

In general, if we can find some effect of a wave which depends similarly on angular orientation, we can conclude that the wave is polarized. Further, we can conclude that the wave must be transverse rather than longitudinal or torsional. Some interesting and important examples of this principle will be presented in Chapter 13.

All three kinds of wave have an important characteristic in common. The disturbances move through the media and away from their sources and continue on their own. We stress this particular characteristic by saying that these waves "propagate," which means we imply more than that they "travel" or "move."

An example will clarify the difference between waves which do propagate and those which do not. Almost every description of the great wheat plains of our middle west, in Canada, or in Central Europe contains a passage about the beautiful wind-formed waves that roll for miles across them. The disturbance is the swaying motion of the wheat. And the regions of that disturbance do indeed travel, but they do not propagate. That is, the disturbance does not originate at a source and then go on by itself but needs to be continually fanned by the wind. When the wind stops, the disturbance stops too. In this regard the traveling "waves" of swaying wheat are not at all the same as our rope and water waves, and we shall therefore concentrate on waves that originate at sources and propagate. For the purposes of this chapter, waves are disturbances which propagate in a medium.

Q1 What kinds of mechanical waves can propagate in a solid?

Q2 What kinds of mechanical waves can propagate in a fluid?

Q3 What kinds of mechanical waves can be polarized?

Q4 Suppose that a mouse runs along under a rug, causing a bump in the rug that travels across the room. Is this moving disturbance a propagating wave?

12.3 Wave propagation. Waves and their behavior may perhaps best be studied by beginning with large mechanical models and focussing our attention on pulses. Consider, for example, a freight train with many cars attached to a powerful locomotive standing still at a railroad crossing. If the locomotive starts abruptly, a disturbance or a displacement wave will be transmitted down the line of cars to the very last one. The shock of the starting displacement proceeds from locomotive to caboose, clacking through the couplings one by one. In this example, the locomotive was the source, the freight cars and their couplings were the medium and the "bump" traveling along the line of cars was the wave. The disturbance proceeds all the way from end to end, and with it goes energy in the form of displacement and motion. Yet no particles of matter are transferred that far; each car only jerks ahead. How much time does it take for the effect of a disturbance created at one point to reach a distant point? The time interval depends upon the speed with which the disturbance or wave propagates. That, in turn, depends upon the type of wave and the characteristics of the medium. In any case, the effect of a disturbance is never transmitted instantaneously. Time is needed for each part of the medium to transfer its energy to the next part.

A very important point: energy transfer can occur without matter transfer.

An engine starting abruptly can start a "bang" wave along a line of cars.

The series of sketches in Fig. 12.5 represent a wave on a rope as seen by a series of frames of a motion picture film, the frames being taken at equal time intervals. The pieces of rope do not travel along with the wave, but each bit of the rope goes through an up-and-down motion while the wave moves to the right. Except at the source of the disturbance on the left, each bit of the rope goes through exactly the same motion as the bit to its left, but its displacement is delayed a moment from the bits closer to the source.

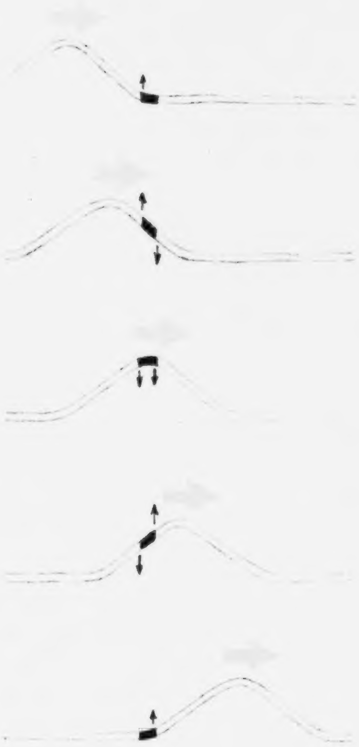


Fig. 12.5 A rough representation of the forces on a small section of rope as a transverse pulse moves past it.

Consider a small section of the rope, labeled X in the diagrams in the margin. When the pulse traveling on the rope first reaches X, the section of rope just ahead of X exerts an upward force on X. As X moves upward, a restoring force pulling X down arises from the next section on. The further up X moves, the greater the restoring forces becomes. So eventually X stops moving up and starts down again. The section of rope ahead of X is now exerting a restoring force and the section behind it is exerting an upward force, so the trip down is similar, but opposite, to the trip up. Finally X is returned to the equilibrium position and both forces vanish.

The time required for X to go up and down that is, the time required for the pulse to pass, depends on two factors: the magnitude of the forces on X and the mass of X. To put it another way, and more generally: the speed with which a wave propagates depends on the stiffness and on the density of the medium. The stiffer the medium, the greater will be the force a section exerts on neighboring sections, and so the greater will be the propagation speed. The greater the density of the medium, the less it will respond to forces, and so the slower will be the propagation. It can be shown that the speed of propagation of a wave depends on the ratio of a stiffness factor and a density factor.

The exact meaning of stiffness and density factors is different for different kinds of waves and different media. For tight strings, for example, the stiffness factor is the tension T in the string and the density factor is the mass per unit length m/ℓ. The propagation speed v is

$$v = \sqrt{\frac{T}{m/\ell}}\ .$$

Q5 What is transferred along the direction of wave motion?

Q6 On what two properties of a medium does wave speed depend?

12.4 Periodic waves. Most of the disturbances we have considered up to now have been short-lived and sudden. The waves set up by single disturbances are called pulses, e.g., the snapping of one end of a rope or the dropping of a stone in a pond, or the sudden bumping of one end of a train. In each case we see a pattern running along the medium with a certain velocity. Continuous regular rhythmic disturbances in a medium result from periodic vibrations which cause periodic waves in that medium. A good example of such a vibration is a swinging pendulum. The swinging is periodic in time, and the pendulum bob executes simple harmonic motion. Another example is the up-and-down motion of a weight at the end of a good spring. The maximum displacement from the position of equilibrium is called the amplitude A and is shown in Fig. 12.6 The time taken to complete one oscillation is called the period and labeled T, while the frequency, or number of vibrations per second, is symbolized by f.

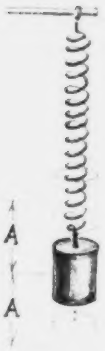


Fig. 12.6

What happens when such a vibration is applied to the end of a rope? Let us think of an experiment where one end of a rope is fastened to an oscillating weight. As the weight vibrates up and down, we observe a wave "traveling" along the length of the rope.

We observe moving crests and troughs along the length of a uniform rope. The source executes simple harmonic motion up and down, and ideally every point along the length of the rope executes simple harmonic motion in turn. The wave travels forward to the right as crests and troughs in turn replace one another, but the points along the rope simply oscillate up and down following the motion of the source. The distance between any two consecutive crests or any two consecutive troughs is always found to be the same along the length of the rope. This distance is called the wavelength of the periodic wave, and is denoted by the Greek letter λ (lambda). As in Fig. 12.7, the amplitude of the wave is represented by A.

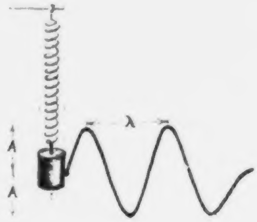


Fig. 12.7

When a single pulse moves from one part of the medium to another, it is fairly clear what is meant by the speed of the pulse. All we need in principle is a clock and a meter stick; watch the front edge of the pulse, and the speed of the pulse is quickly found. But what if we cannot observe the source or the beginning or ending of a wave train? We will show that the speed of a periodic wave can be easily found from its frequency and wavelength.

As a wave progresses, each point in the medium oscillates periodically with the frequency and period of the source. Figure 12.8 illustrates a periodic wave moving to the right, frozen every 1/4 period. Follow the progress of the crest that started out at the extreme left. The time it takes this crest to move a distance of one wavelength is the time it takes a point in the medium to go through one complete oscillation. That is, the crest moves one wavelength λ in one period of oscillation T. The speed of the crest is therefore

$$v = \frac{\text{distance travelled}}{\text{time taken}} = \frac{\lambda}{T} \, .$$

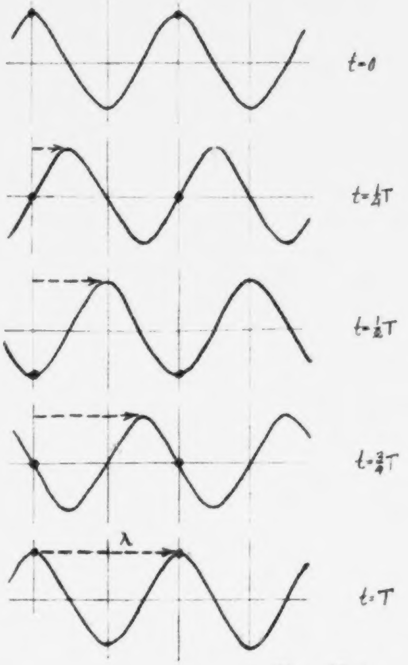


Fig. 12.8

The speed of this wave is the same thing as the speed of any one of its crests. We say, therefore, that the speed of the wave,

$$v = \frac{\text{wavelength}}{\text{period of oscillation}} = \frac{\lambda}{T} .$$

But $T = \frac{1}{f}$, where f = frequency (see Unit 1, Chapter 4, page 106). Therefore $v = f\lambda$, or wave speed = frequency × wavelength.

The wave generated by a simple harmonic vibration is a sine wave. It has the same shape as a graph of the sine function familiar in trigonometry.

We can also write this relationship as $\lambda = \frac{v}{f}$ or $f = \frac{v}{\lambda}$. These expressions imply that, for waves of the same speed, the frequency and wavelength are inversely proportional—a wave of twice the frequency would have only half the wavelength, and vice versa. This inverse relation of frequency and wavelength will be useful in other units in this course.

In Fig. 12.9, sets of points are marked which are in step. The crest points C and C′ have reached maximum displacement positions in their vibrations in the upward direction. The trough points D and D′ have reached maximum displacement positions in the downward direction. The points C and C′ have been chosen such that they have identical displacements and velocities at any instant of time. Their vibrations are identical and in unison. The same is true for the points D and D′. Indeed there are indefinitely many such points along the length of the wave which are vibrating identically. Note that C and C′ by definition are a distance λ apart, and so are D and D′. Points such as C and C′ are said to be in phase with one another as are also points D and D′. Points separated from one another through distances of λ, 2λ, 3λ,... nλ, are all in phase with one another. These can be anywhere along the length of the wave and need not correspond with only the high or low points. For example, points such as P, P′, P′′, are all in phase with one another. They are each separated from the other by the distance λ.

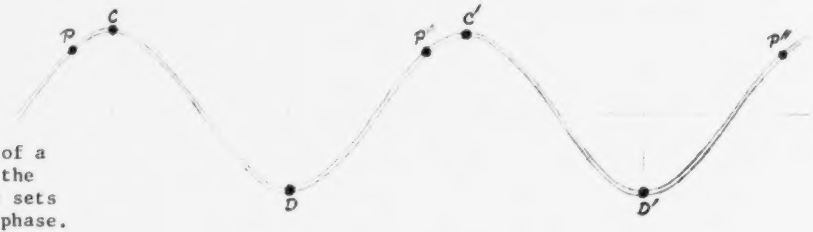


Fig. 12.9 A "snapshot" of a periodic wave moving to the right. Letters indicate sets of points with the same phase.

Some of the points are exactly out of step. Point C reaches its maximum upward displacement at the same time that D reaches its maximum downward displacement. At the instant that C begins to go down, D begins to go up. Points such as these are called one-half wave out of phase with respect to one another; C and D′ also are one-half wave out of phase. Points separated from one another through distances of $\frac{\lambda}{2}$, $\frac{3\lambda}{2}$, $\frac{5\lambda}{2}$,... are one-half wave out of phase.

Q7 Of the wave variables—frequency, wavelength, period, amplitude and polarization—which ones describe
1) space properties of waves?
2) time properties of waves?

Q8 How can "wavelength" be defined for a wave that isn't a regular sine wave?

Q9 A vibration of 100 cycles per second produces a wave.
1) What is the wave frequency?
2) What is the period of the wave?
3) If the wave speed is 10 meters per second, what is the wavelength? (You can look back to find the relationship you need to answer this.)

Q10 If points X and Y on a periodic wave are one-half wave "out of phase" with each other, which of the following must be true?
a) X oscillates at half the frequency at which Y oscillates.
b) X and Y always move in opposite directions.
c) X is a distance of one-half wavelength from Y.

12.5 When waves meet: the superposition principle. So far we have considered single waves in isolation from other waves. What happens when two waves encounter each other in the same medium? Suppose that two waves approach each other on a rope, one traveling to the right and one traveling to the left. The series of sketches in Fig. 12.10 show what happens. The waves pass through each other without either being modified. After the encounter, each wave shape looks just as it did before and is traveling along just as it was before. This phenomenon of passing through each other unchanged is common to all types of waves. You can easily see that it is true for surface ripples on water. (Look back, for example, to the opening photograph for the chapter.) You could infer that it must be true for sound waves by recalling that two conversations can take place across a table without either distorting the other.

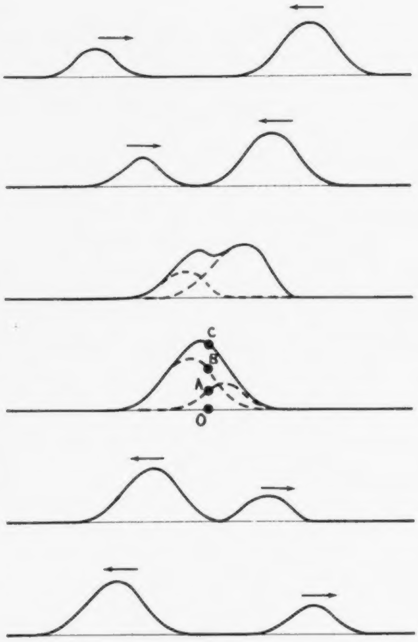


Fig. 12.10 The superposition of two rope waves. The dashed curves are the contributions of the individual waves. As an example, in the fourth sketch the displacement OA plus the displacement OB gives the resulting displacement of the rope OC.

But what is going on during the time when the two waves overlap? They add up. At each instant the rope's displacement at each point in the overlap region is just the sum of the displacements that would be caused by each of the two waves alone. If two waves travel toward each other on a rope, one having a maximum displacement of 1 cm upward and the other a maximum displacement of 2 cm upward, the total maximum upward displacement of the rope while these two waves pass each other is 3 cm.

What a wonderfully simple behavior, and how easy it makes everything! Each wave proceeds along the rope making its

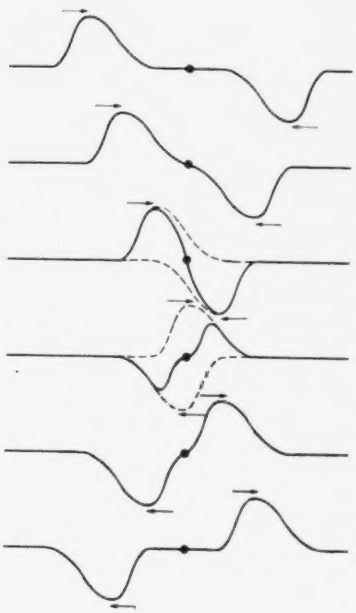


Fig. 12.11

own particular contribution to the rope's displacement no matter what any other wave may be doing. If we want to know what the rope looks like at any instant, all we need do is add up the displacements due to each wave at each point along the rope. We say that waves obey a superposition principle. Figure 12.11 shows another case of wave superposition. Notice that when the displacements are in opposite directions, they tend to cancel each other. This still fits the addition rule, since one direction of displacement is considered negative.

The superposition principle applies no matter how many separate waves or disturbances are present in the medium. The examples shown in Figs. 12.10 and 12.11 illustrate the principle applied when only two waves are present, but we can discover by experiment that the superposition is just as valid when there are three, ten, or any number of waves. Each wave makes its own contribution and the net result is simply the sum of all the individual contributions.

This simple additive property of waves permits us to add waves graphically. You should check the diagrams with a ruler to see that the net displacement (full line) is just the sum of the individual displacements (dashed lines) in these two cases.

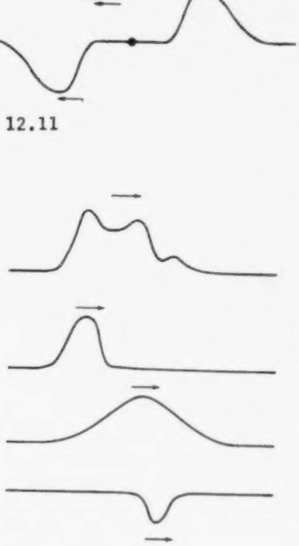


Fig. 12.12

We can turn the superposition principle around. If it is true that waves add as we have described, then we can think of a complex wave as the sum of a set of simpler waves. In Fig. 12.12 at the left, a complex wave has been analyzed into a set of three simpler waves.

The French mathematician Jean-Baptiste Fourier announced a theorem in 1807 that any periodic oscillation, regardless of its complexity, can be analyzed as the sum of a series of simpler regular wave motions. Fourier was interested in the theory of heat, sound, light and electricity, and his theorem became a basic tool for harmonic analysis in all these areas. Fourier showed the general validity of the superposition principle.

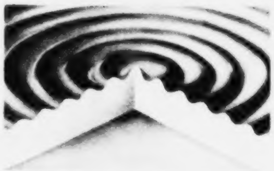
Q11 Two periodic displacement waves of amplitudes A_1 and A_2 are passing through a point P. What will be the greatest displacement of point P?

Q12 The superposition principle states that wave amplitudes add. How then can waves cancel each other out?

Jean-Baptiste Fourier (1768-1830), a tailor's son who became a brilliant mathematician in Napoleonic France.

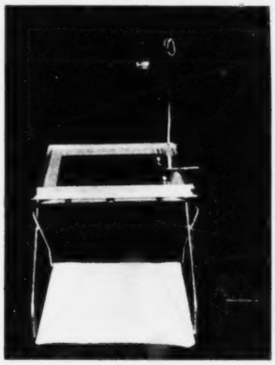
12.6 A two-source interference pattern. We can see the superposition principle at work in the important wave phenomenon known as interference. Figure 12.13 is a photograph of ripples spreading away from a small sphere, vibrating up and down into the water surface. What we see here is the spatial pattern of the water level at an instant. Fig. 12.14 is a photograph of the water surface when it is agitated by two vibrating spheres. The two small sources go through their up and down motions together, that is, the sources are in phase. The photograph catches the pattern of the overlapping waves at one instant, called an interference pattern.

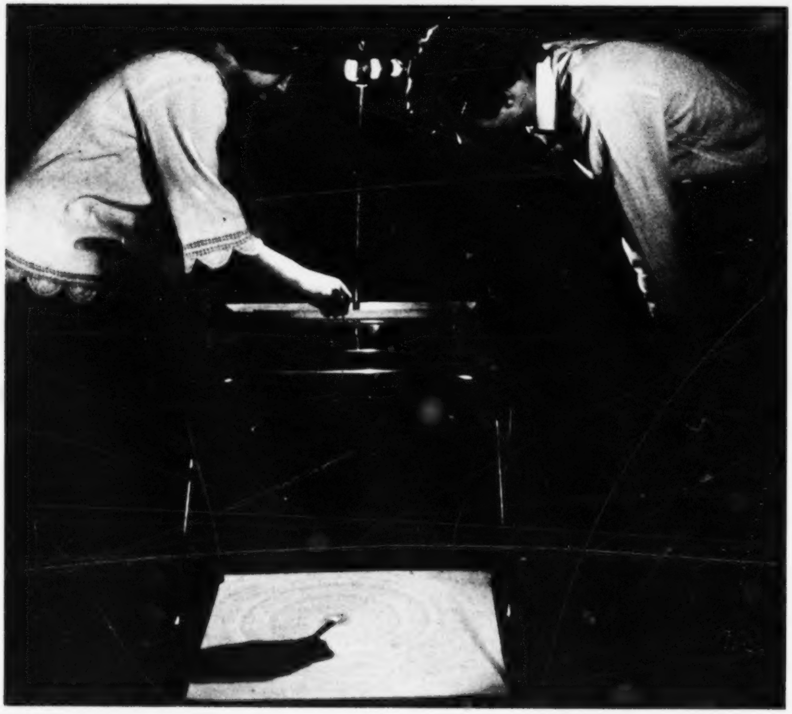
Can we interpret what we see in this photograpn in terms of what we already know about waves? And can we describe how the pattern will change with time? If you tilt the page so that you are viewing Fig. 12.14 from a glancing direction, you will see more clearly some spokes or strips of intermediate shade—neither as bright as the crest, nor as dark as the trough of waves. This feature can be easily explained by the superposition principle.



Drawing of surface wave on water.

A "ripple tank" set-up for viewing the behavior of waves.





The ripple tank, being used here by students to observe a circular pulse, can be fitted with vibrator to produce periodic wavetrains. Fig. 12.13 is an instantaneous photograph of the shadows of ripples produced by a vibrating point source. For Fig. 12.14 there were two point sources vibrating in phase.

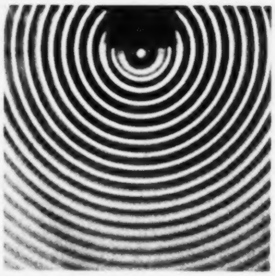


Fig. 12.13

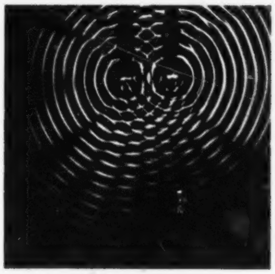
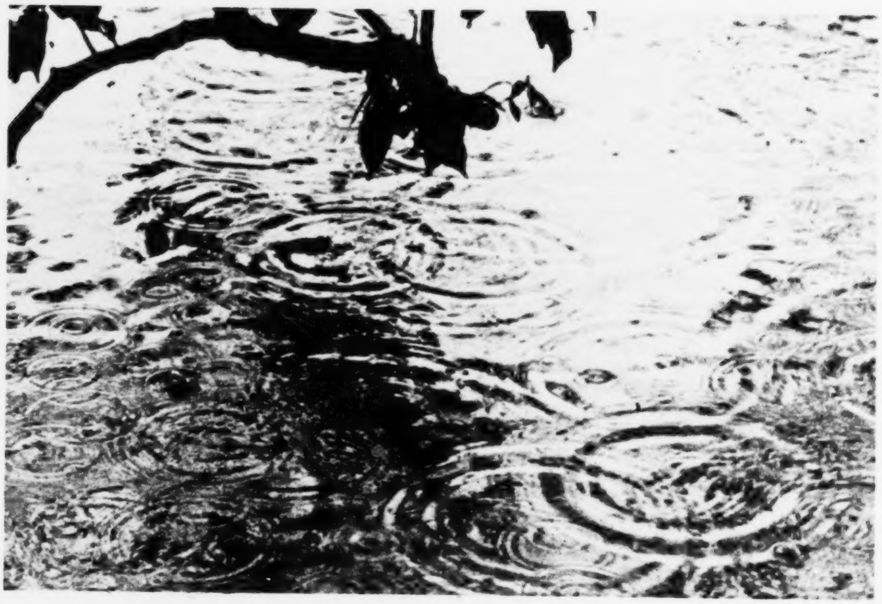


Fig. 12.14



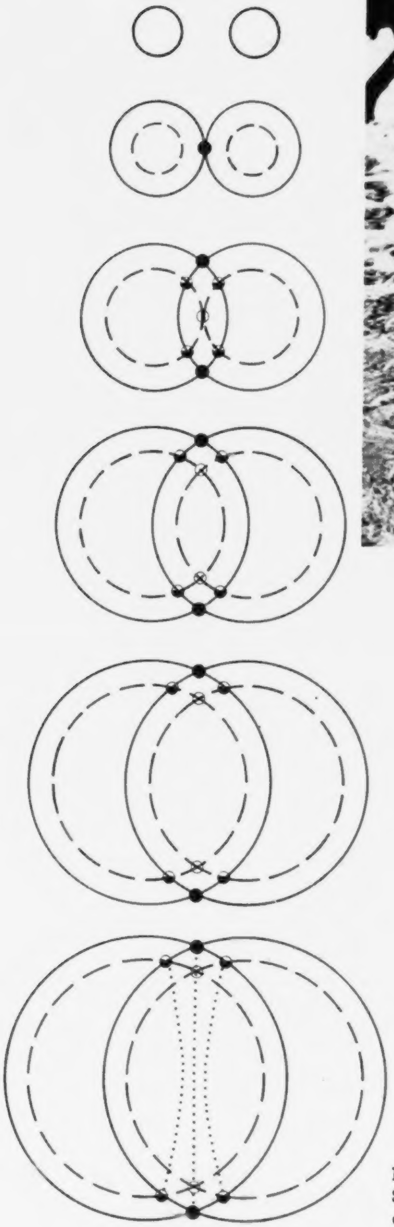
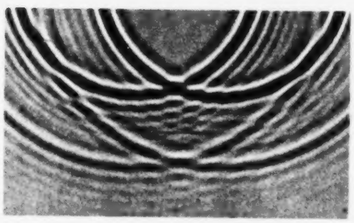


Fig. 12.15
Superposition of
circular pulses.



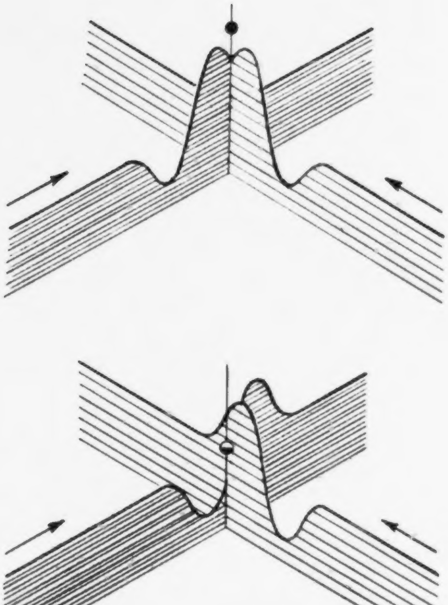


Fig. 12.16 Diagram representing the superposition of pulses in Fig. 12.15. In (a) two crests are arriving at the vertical line. In (b) a crest is arriving together with a trough. The dark and half-dark balls show the net displacement.

Suppose that two sources produce identical pulses at the same instant, each pulse containing one crest and one trough as shown in Fig. 12.15. The height of each crest above the undisturbed level is equal to the depth of each trough below. The sketches show the patterns of the water surface after equal time intervals. As the pulses spread out, the points at which they overlap move too. In the figure we have placed a completely darkened circle wherever a crest overlaps another crest, a blank circle wherever a trough overlaps another trough, and a half-darkened circle wherever a crest overlaps a trough. Applying the superposition principle to this situation, we conclude that the water level is highest at the completely darkened circles, lowest at the blank circles, and at the equilibrium height at the half-darkened circles. Each of the sketches in Fig. 12.15 represents the spatial pattern of the water level at an instant. The dotted curves in the last sketch in Fig. 12.15 are the paths followed by the overlap regions during the time covered by the earlier sketches.

At points in Fig. 12.15 which are marked with darkened and blank circles the two pulses arrive in phase, as shown in Fig. 12.16(a). The waves reinforce each other causing a greater amplitude and are thus said to *interfere constructively*. All such points are at the same distance from each source.

At the points in Fig. 12.15 marked with half-darkened circles, the two pulses arrive completely out of phase, as shown in Fig. 12.16(b). Here the waves are said to interfere *destructively*, leaving the water surface undisturbed. All such points are one crest-trough distance further from one source than from the other.

Now we can interpret the photograph of Fig. 12.14. The centers of what we called "strips of alternating character" are areas where waves cancel or reinforce each other, called *nodal* or *antinodal lines*, respectively. Look closely at Fig. 12.17 and notice its symmetry. The central spoke or strip labeled A_0 is an antinode where reinforcement is complete. As the waves spread out, points on these lines are displaced up and down *much more* than they would due to either wave alone. The outside nodes labeled N_4 represent lines where destructive interference is at a maximum. As the waves spread out, points on these lines move up and down *much less* than they would due to either wave alone. You should compare the drawing in Fig. 12.17 with the photograph in Fig. 12.14 to make sure you know which are the antinodal lines and which are the nodal lines.

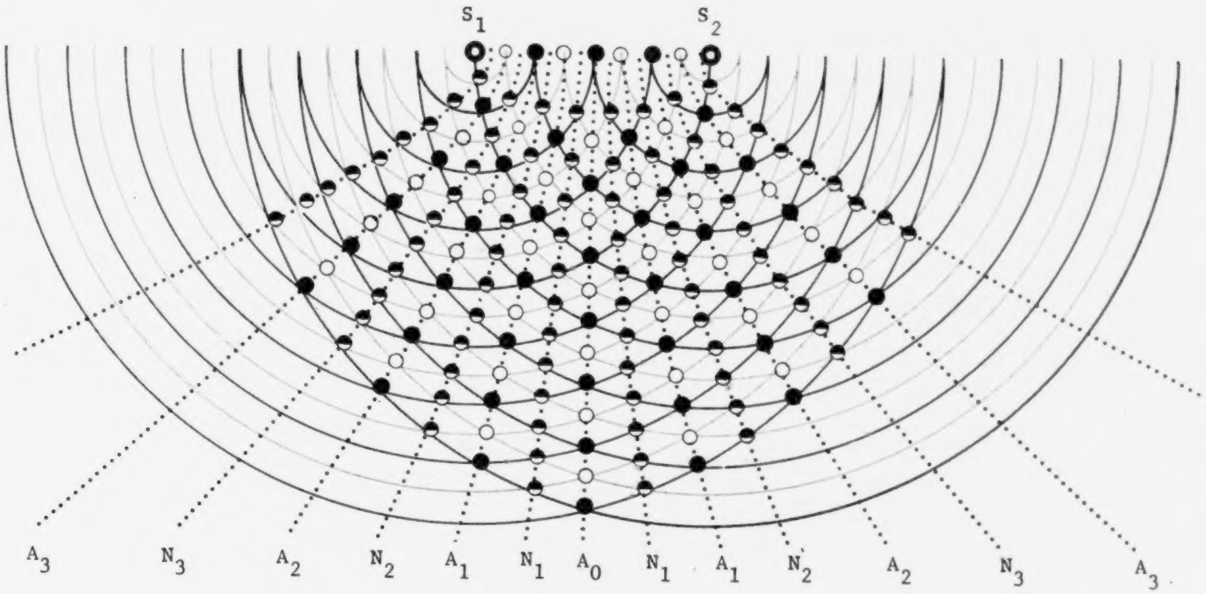


Fig. 12.17 A "snapshot" diagram of the interference pattern due to two in-phase periodic sources separated by four wavelengths. The letters A and N designate antinodal and nodal lines. The dark circles indicate where crest is meeting crest, the blank circles where trough is meeting trough, and the half-dark circles where crest is meeting trough.

If we know the wavelength λ and the source separation, then we can calculate the angles that these lines make with the central line A_0. Or, if we know these angles and the source separation, then we can calculate the wavelength. The observation of a two-source interference pattern allows us to make a wavelength calculation, even if we are unable to see the waves directly! For example, an interference pattern in sound can be created with two loudspeakers being driven in phase, and nodal and antinodal points can be located by ear. But if you do the experiment indoors, you may discover unexpected nodal points or "dead spots" caused by reflections from walls or other obstacles.

What determines the positions of the nodal and antinodal lines? Figure 12.18 shows part of the pattern of Figure 12.17. At any point P on an antinodal line, the waves from the two sources arrive in phase. This occurs where P is some whole number of wavelengths further from one source than from the other; that is, where the difference in distance $\overline{S_2P} - \overline{S_1P} = n\lambda$, λ being the wavelength and n being any whole number (including zero). At any point Q on a nodal line, the waves from the two sources arrive exactly out of phase. This occurs where Q is some odd number of half-wavelengths ($\frac{1}{2}\lambda$, $\frac{3}{2}\lambda$, $\frac{5}{2}\lambda$, etc.) further from one source than from the other; that is, where $\overline{S_1Q} - \overline{S_2Q} = (n + \frac{1}{2})\lambda$. If the distance ℓ from midway between the sources to a detection point is much larger than the source separation d, so that the point lies on the relatively straight part of the nodal or antinodal line, then there is a simple relationship between the node position, the wavelength λ and source separation d. The details of the relationship and the calculation of wavelength are described on the next page.

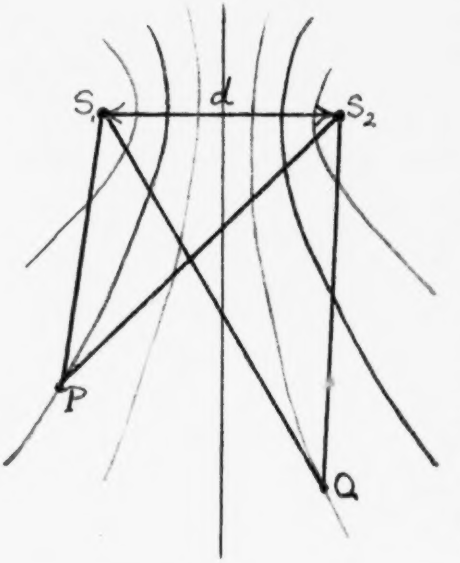


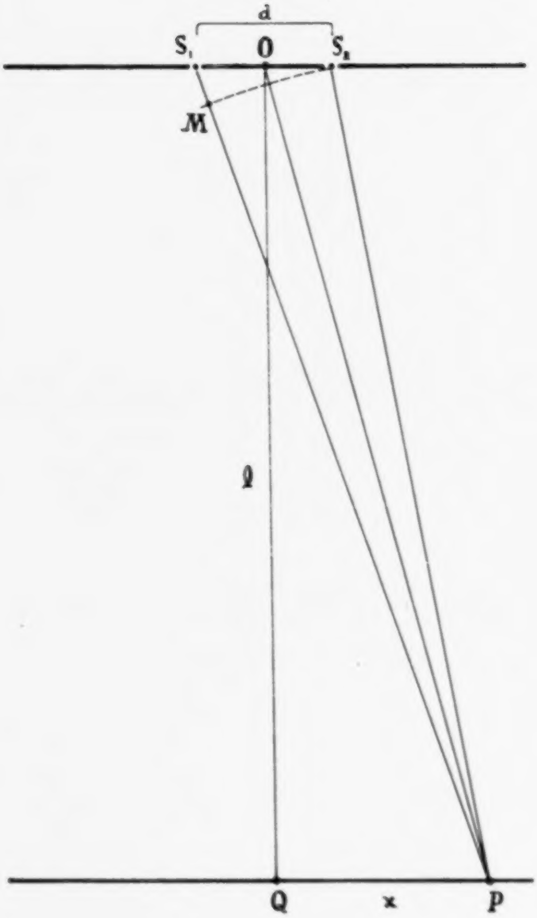
Fig. 12.18

Calculating Wave Lengths from an Interference Pattern

d = separation between sources S_1 and S_2 (they may be two sources that are truly in phase, or two slits through which a previously prepared wave front passes)

ℓ = distance from sources to detection line parallel to the two sources

x = distance from center line to point P along the detection line



Waves reaching P from S_1 travel farther than waves reaching P from S_2. If the extra distance is λ (or 2λ, 3λ, etc.), the waves will arrive in phase and P is a point of strong wave disturbance. If the extra distance is $\frac{1}{2}\lambda$ (or $\frac{3}{2}\lambda$, $\frac{5}{2}\lambda$, etc.), the waves will arrive out of phase and P is a point of weak wave disturbance.

With P as center we draw an arc of a circle of radius $\overline{PS_2}$; it is indicated on the figure by the dotted line $\overline{S_2M}$. Then line $\overline{PS_2}$ = line $\overline{PM}$ and therefore the extra distance that the wave from S_1 travels is the length of the segment $\overline{S_1M}$.

Now if d is very small compared to ℓ, the circular arc $\overline{S_2M}$ is a very small piece of a large circle and is practically the same as a straight line. Also the angle S_1MS_2 is very nearly 90°, so that triangle S_1S_2M can be regarded as a right triangle. Furthermore angle S_1S_2M is equal to angle POQ. Then right triangle S_1S_2M is similar to right triangle POQ.

So

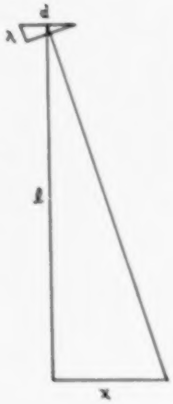
$$\frac{\overline{S_1M}}{\overline{S_1S_2}} = \frac{x}{\overline{OP}}. \qquad (12.1)$$

But $\overline{S_1M}$ is the extra distance traveled by the wave from source S_1. For P to be a maximum of disturbance, S_1M must equal λ (or 2λ, 3λ, etc.), $\overline{S_1S_2}$ is the source separation d. OP is very nearly equal to ℓ. Then Eq. (12.1) becomes

$$\frac{\lambda}{d} = \frac{x}{\ell} \qquad (12.2)$$

and

$$\lambda = \frac{d}{\ell}\,x \qquad (12.3)$$



If we measure the source separation d, the distance ℓ, and the distance x to the first disturbance maximum beside the center, we can calculate the wavelength from Eq. (12.3).

This analysis allows us to calculate the wavelength of any wave phenomenon, whether it is water ripples, sound, light, etc.; it will therefore be found very useful in later Units.

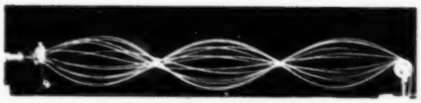
Q13 Are nodal lines in interference patterns regions of cancellation or regions of reinforcement?

Q14 What are antinodal lines?

Q15 Nodal points in an interference pattern occur where
- a) the waves arrive "out of phase."
- b) the waves arrive "in phase."
- c) the point is equidistant from the wave sources.
- d) the point is one-half wavelength from both sources.

Q16 Under what circumstances do waves from two in-phase sources arrive at a point out of phase?

12.7 Standing waves. If both ends of a rope or spring are shaken very carefully, with the same frequency and same amplitude, a very interesting phenomenon can be produced. The interference of the identical waves coming from both ends will result in certain points on the rope not moving at all! In between these nodal points, the rope oscillates back and forth with no apparent propagation of wave patterns. This phenomenon is called a "standing wave" or "stationary wave" and is pretty to watch. (Using the superposition principle, you can show that this is just what would be expected from the addition of the two oppositely traveling waves.)



A vibrator at the left produces a wave train that runs along the rope and reflects from the fixed end at the right. The sum of the oncoming and the reflected waves is a standing wave pattern. This stroboscopic photograph shows 12 instantaneous positions of the rope.

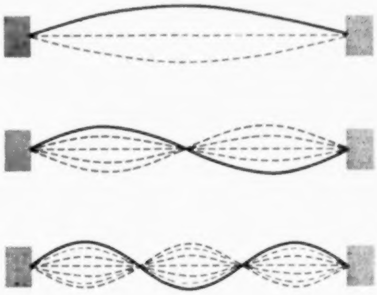
The same effect can be produced by the interference of a continuous wave with its reflection. To make standing waves, there do not have to be two people shaking the opposite ends of the rope; one end can be tied to a hook on a wall. The train of waves sent down the rope by shaking one end will reflect back from the fixed hook and interfere with the new, oncoming wave and can produce the standing pattern of nodes and oscillation. In fact, you can go further and tie both ends of the rope to hooks and pluck or hit the rope. From the plucked point a pair of waves go out in opposite directions and then reflect back and forth from the fixed hooks. The interference of these reflected waves traveling in opposite directions can produce a standing pattern just as before. Standing waves on guitars, violins, pianos and all other stringed instruments are produced in just this fashion. Because the vibrations of strings are in standing waves, the vibration frequencies depend on the speed of wave propagation along the string and the length of the string.

Painting on a Greek vase from 5th century B.C.

Museum of Fine Arts, Boston

The connection between length of string and musical tone, recognized over two thousand years ago, was of the greatest importance in contributing to the idea that nature is built on mathematical principles. When strings of equal tautness and diameter are plucked, pleasing harmonies result if the lengths of the strings are in the ratios of small whole numbers. Thus the ratio 2:1 gives the octave, 3:2 the musical fifth and 4:3 the musical fourth. This striking connection between music and numbers encouraged the Pythagoreans to search for other numerical harmonies in the universe. The Pythagorean ideal strongly affected Greek science and became an inspiration for much of Kepler's work. In a generalized form, the ideal flourishes to this day in many beautiful applications of mathematics to physical experience.

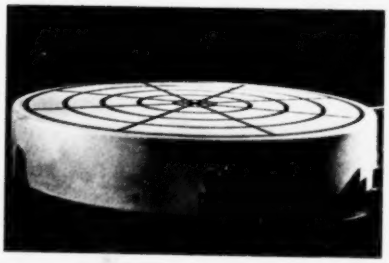
Using nothing more than the superposition principle, we can now state the harmonic relationship much more precisely. Standing patterns can be produced by reflections from the boundaries of a medium only for certain wavelengths (or frequencies). In the example of a string fixed at both ends, the two ends are fixed and so must be nodal points. Thus the longest traveling waves that can set up standing waves on a rope will be those for which one-half wavelength just fits on the rope. Shorter waves can produce standing patterns with more nodes, but only when some number of one-half wavelengths just fit on the rope. The shorter wavelengths correspond to higher frequencies, so the principle can be stated that on any bounded medium, only certain frequencies of standing waves can be set up. On an idealized string, there are in principle an unlimited number of frequencies, all simple multiples of the lowest frequency. That is, if f_0 is the lowest possible frequency of standing wave, the other possible standing waves would have frequencies $2f_0$, $3f_0$,.... These higher frequencies are called "overtones" or "harmonics" of the "fundamental" frequency f_0.



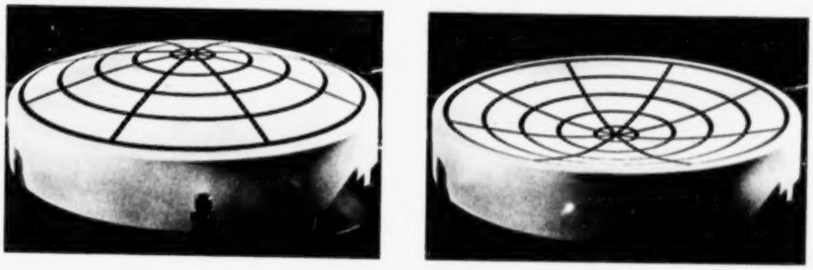
In real media, there are practical upper limits to the possible frequencies and the overtones are not exactly simple multiples of the fundamental frequency (that is, the overtones are not strictly harmonic). In more complicated systems than stretched strings, like the enclosure of a saxophone, the overtones may not be even approximately harmonic. As you might guess from the superposition principle, standing waves of different frequencies can exist in a medium at the same time. A plucked guitar string, for example, will oscillate in a pattern which is the superposition of the standing waves of many overtones. The relative oscillation energies of the different overtones of string instruments or in the enclosure of horns and organ pipes determine the "quality" of the sound they produce. The difference in the balance of overtones is what makes the sound of a violin distinct from the sound of a trumpet, and both distinct from a soprano voice, even if all these are sounding with the same fundamental frequency.

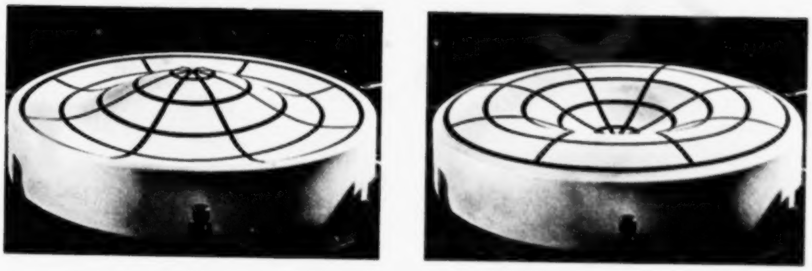
Q17 When two identical waves of frequency f, traveling in opposite directions, interfere to produce a standing wave, what is the motion of the medium at

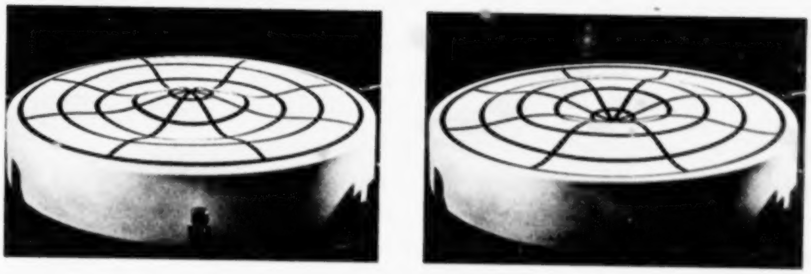
1) the nodes of the standing wave?
2) the antinodes (loops) of the standing wave?

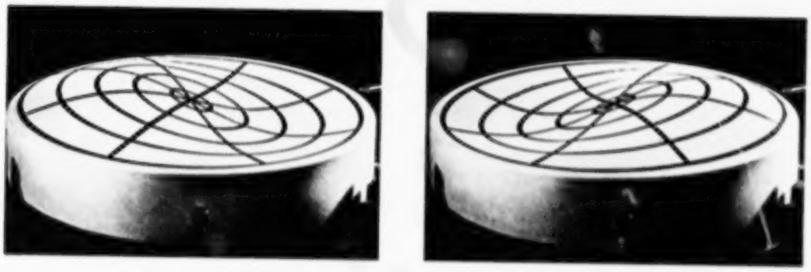


A rubber "drumhead" first at rest, then made to vibrate in each of four of its many possible modes.









Q18 If the two interfering waves have wavelength λ, what is the distance between the nodal points of the standing wave?

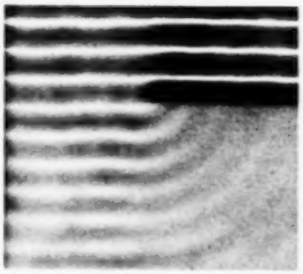
Q19 What is the wavelength of the longest traveling waves which can produce a standing wave on a string of length L?

Q20 Can standing waves of any frequency, as long as it is higher than the fundamental, be set up in a bounded medium?

12.8 Wave fronts and diffraction.

Waves go around corners. We are so used to sound waves doing this that we scarcely notice it. This phenomenon of the energy of waves spreading into what we would expect to be "shadow" regions is called diffraction.

Once again we turn to water waves to see this point best. From among all the arrangements of barriers that can result in diffraction, we will concentrate on two. The first of these is shown in Fig. 12.20, which is a photograph of straight water waves being diffracted through a narrow slit in a straight barrier. The barrier is parallel to the wave's crest lines, and the slit is less than one wavelength wide. Figure 12.21 is a photograph of the second barrier arrangement we want to investigate. There are two narrow slits in the barrier, and the pattern of the diffracted wave is the same as that given by two point sources which are in phase. We get the same kind of result no matter how many narrow slits we put in the barrier placed parallel to the incoming wave. That is, the pattern of the diffracted wave is just that which would be produced if a point source were put at the center of each slit position, with such sources in phase.



Diffraction of ripples around the edge of a barrier.

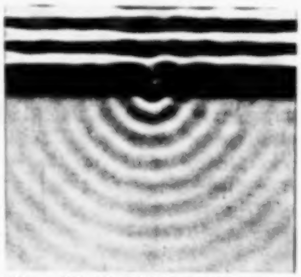


Fig. 12.20 Diffraction of ripples through narrow opening.

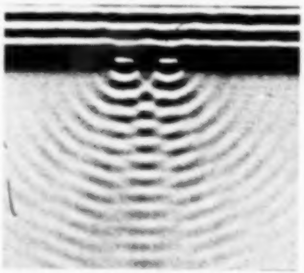


Fig. 12.21 Diffraction of ripples by two narrow openings. The diffracted waves spread out and overlap, producing the same interference effect as we studied for two actual sources in Section 12.6.

We can describe these and all other diffraction patterns if we understand a characteristic of waves, first enunciated by Christiaan Huygens in 1678 and now known as Huygens' principle. Before giving a statement of the principle, we need the definition of a wave front.

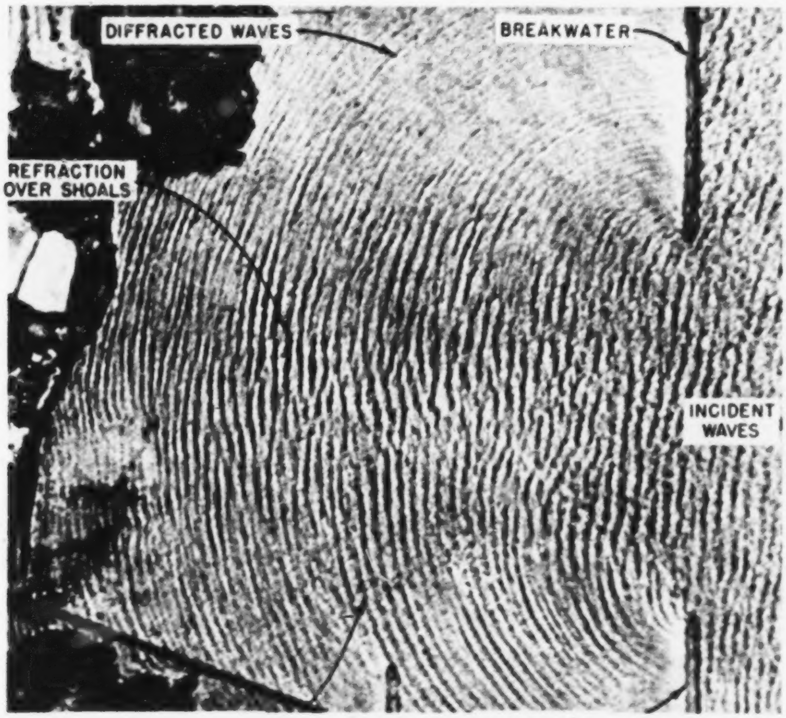
What we have called crest lines and trough lines of water waves are special cases of wave fronts. For a water wave, a wave front is an imaginary line along which every point on the water's surface is in exactly the same stage of its vibrational motion, that is, in the same phase. Crest lines are wave fronts, since every point on the water's surface along a crest line has just reached its maximum displacement upward, is momentarily at rest, and will start downward an instant later. For "straight-line" water waves, all wave fronts are straight lines parallel to each other. For circular waves, the wave fronts are all circles.

For sound waves created by a handclap, the wave fronts become at a distance very nearly spherical surfaces. At large distances from such a source of sound, i.e., where the radii of the spherical wave fronts are large, a small section of the wave front is nearly flat. Thus circular or spherical wave fronts become virtually straight-line or flat-plane wave fronts at great distances from their sources.

Now here is Huygens' principle as it is generally stated today: every point on a wave front may be considered to behave as a point source for waves generated in the direction of the wave's propagation. As Huygens said:

> There is the further consideration in the emanation of these waves, that each particle of matter in which a wave spreads, ought not to communicate its motion only to the next particle which is in the straight line drawn from the luminous [source] point, but that it also imparts some of it necessarily to all others which touch it and which oppose themselves to its movement. So it arises that around each particle there is made a wave of which that particle is the center.

The one-slit and two-slit diffraction patterns are certainly consistent with Huygens' principle. In the two-slit case (Fig. 12.21), each wave front arrives at the two slits at the same time, so the oscillations in the slit are in phase. That is why the interference pattern produced by the waves diffracted through these slits matches that produced by two point sources which oscillate in phase.



Diffraction of ocean waves at a breakwater.

We can understand all diffraction patterns if we keep both Huygens' principle and the superposition principle in mind. For example, consider a slit, wider than one wavelength, in which case the diffraction pattern contains nodal lines. Figure 12.22 shows why nodal lines appear. There surely exist points like P that are just λ farther from one side of the slit A than from the other side B. In that case, AP and OP differ by one-half wavelength λ/2. In keeping with Huygens' principle, we imagine the points A and O to be in-phase point sources of circular waves. But since AP and OP differ by λ/2, the two waves will arrive at P completely out of phase. So, according to the superposition principle, the waves from A and O will cancel at point P.

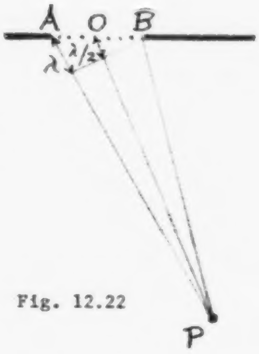


Fig. 12.22

But the argument made for the pair of "sources" at A and O can also be made for the pair consisting of the first point to the right of A and the first to the right of O. In fact, the same is true for each such matched pair of points all the way across the slit. Since the waves originating at each pair cancel, the point P is a nodal point on a nodal line. Then we should see nodal lines in the diffraction pattern of a slit wider than λ, and we do, as Fig. 12.23 shows. If the slit width is less than λ, then there can be no nodal point, since no point can be a distance λ farther from one side of the slit than from the other. Slits of widths less than λ behave nearly as point sources. The narrower they are, the more nearly their behavior is that of point sources.

We can measure the wavelength of a wave by investigating the interference pattern where the diffracted waves overlap. For example, we can analyze the two-slit pattern (Fig. 12.21) in just the same way we analyzed the two-source interference pattern in Sec. 12.6.

The larger the wavelength compared to the distance between the slits, for two-slit interference, the more the interference pattern spreads out. That is, as λ increases or d decreases, the nodal and antinodal lines make increasingly large angles with the straight-ahead direction. Similarly, for single-slit diffraction, the pattern spreads when the ratio of wavelength to the slit width increases. For a given slit width, the longer wavelength diffraction is the more pronounced. That is why, when you hear a band playing around a corner, you hear the bass drums and tubas better than the piccolos and cornets even if they have equal energy output.

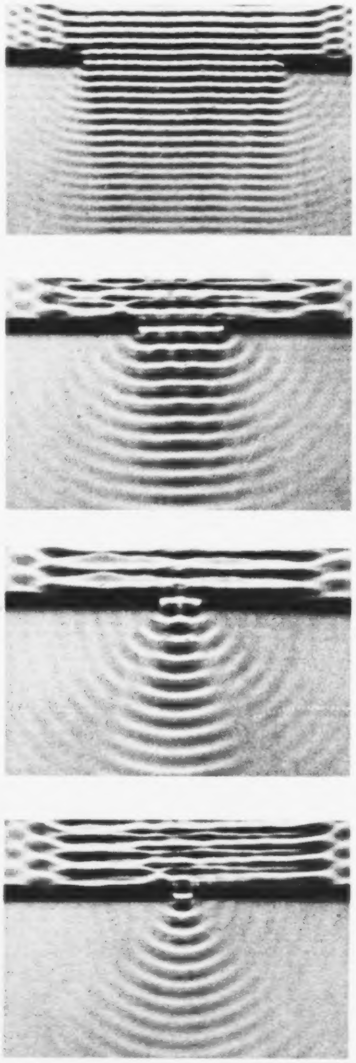


Fig. 12.23

Sound waves have been understood to behave according to these rules of interference and diffraction for a long time, but light, as we shall see in Chapter 13, was only proven to exhibit interference and diffraction after 1800. The reason for this delayed discovery is primarily the fact that diffraction effects are much less for very short wavelengths. Measurements of light had to be extraordinarily accurate to show that light can bend around corners and can be made to interfere. These properties of light seem contrary to everyday experience, in contrast to the next two properties of wave motion we shall consider, namely reflection and refraction.

Q21 What characteristic do all points on a wave front have in common?

Q22 State Huygens principle.

Q23 Why can't there be nodal lines in a diffraction pattern from an opening less than one-half wavelength wide?

Q24 What happens to the diffraction pattern from an opening as the wavelength increases?

Q25 Diffraction of light went unnoticed for centuries because
- a) light travels so fast.
- b) light has such a short wavelength.
- c) light was seldom sent through holes.
- d) light waves have a very small amplitude.

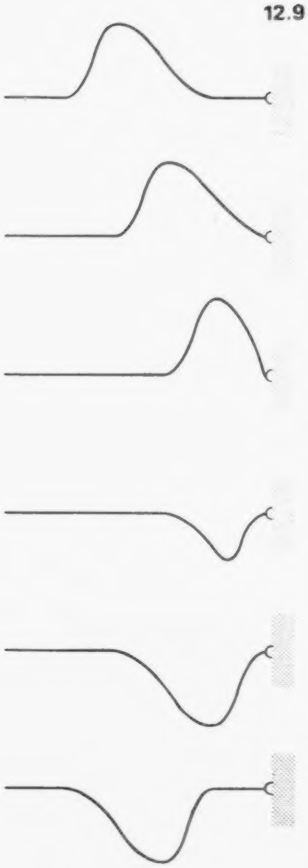


Fig. 12.24

12.9 Reflection. In addition to passing through one another and spreading around obstacles in their paths, waves also bounce away wherever they reach any boundary to their media. Echoes are familiar examples of the reflection of sound waves. The property of reflection is shared by all waves, and again the superposition principle, as well as Huygens' principle, will help us understand what is happening when reflection occurs.

We send a wave down the rope toward an end which is tied to a massive solid object, such as a wall in a building. The force that the wave exerts at the rope's end cannot do any work when the end does not move. If the rope is tied tightly to a hook securely fastened to a massive wall, the energy carried in the wave will not be absorbed at the rope's end. Then the wave will bounce back or be reflected, and ideally it will continue to carry the same energy.

What will the wave look like after it is reflected? The striking result is that the wave is flipped upside down on reflection. As the wave comes in from left to right and encounters the fixed hook, the hook must exert a force on the rope while the reflection is taking place. The description of the detailed way in which that force varies in time is complicated. The net effect is that an inverted wave of the same form is sent back down the rope.

So far we have been dealing with reflections of one-dimensional waves. If we now turn our attention to the two-dimensional water surface waves, we can have variously shaped crest lines, variously shaped barriers from which to get reflections, and various directions in which the waves can approach the barrier.

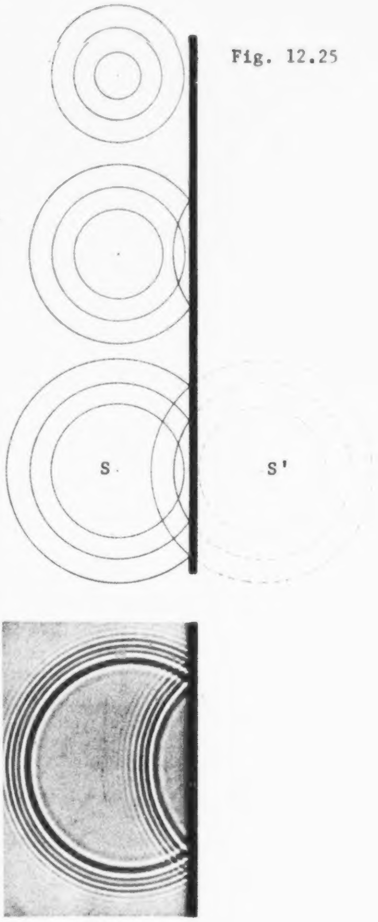


Fig. 12.25

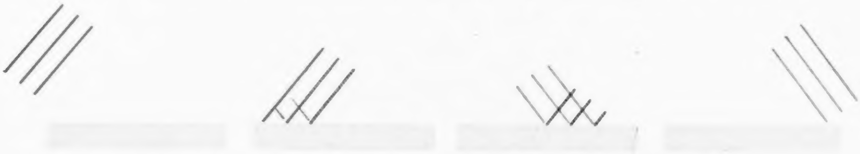
If you have never watched closely as water waves are reflected from a fixed barrier, you should do so before another day passes. Any still pool or water-filled wash basin or tub will do to watch the circular waves speed outward, reflect from rocks or walls, run through each other, and finally die out.

Dip your finger tip into and out of the water quickly, or let a drop of water fall from your finger into the water. Now watch the circular wave approach and then bounce off a straight wall or a board. The long side of a tub is fine as a straight barrier.

Figure 12.25 pictures what you will see, where S is the point of the waves' source. Three crests are shown in the sketches. You may see more or fewer than three good ones, but that does not matter. In the upper sketch, the outer crest is approaching the barrier at the right. The next two sketches show the positions of the crests after first one and then two of them have been reflected. The dashed curves in the last sketch are an attempt to show that the reflected wave appears to originate from a nonexistent source S′ behind the barrier, S′ being as far behind the barrier as S is in front of it. The imaginary source at the point S′ is called the image of the source S.

We looked at the reflection of circular waves first, because that is what one is likely to notice first when studying water waves. But it is easier to see a general principle in operation when we look at a straight wave front reflecting from a straight barrier.

If we push a half-submerged ruler or any straightedge quickly back and forth parallel to the water surface, we will generate a wave in which the crests lie along straight lines over a large fraction of the ruler's length. Figure 12.26 (a) shows what happens when this wave reflects obliquely from a straight barrier. The first sketch shows three crests approaching the barrier; the last shows the same crests as they move away from the barrier after the encounter. The two sketches between show the reflection process at two different intermediate instants.



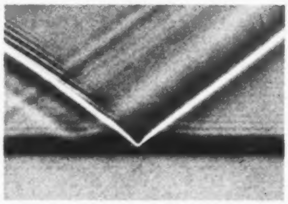


Fig. 12.26 (a)

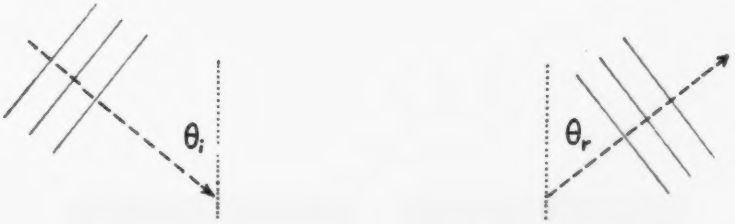


Fig. 12.26(b)

The sketches in Fig. 12.26 (b) include dashed construction lines which were drawn so that they are perpendicular to the wave's crest lines. Imaginary lines of this kind are called rays, and they are often helpful when describing wave behavior. The important feature of a ray is that the wave's velocity at any point lies along the ray at that point.

The behavior of the wave upon reflection, as pictured in Fig. 12.26, is described easily in terms of rays. As shown in the first sketch, a representative ray for the incident wave makes the angle θ_i with a line drawn perpendicular to the reflecting surface. (The perpendicular is the dotted line in the figure.) A representative ray for the reflected wave makes the angle θ_r with the same perpendicular line. The experimental fact is that these two angles are equal; the angle of incidence θ_i is equal to the angle of reflection θ_r. You can prove it for yourself in the laboratory.

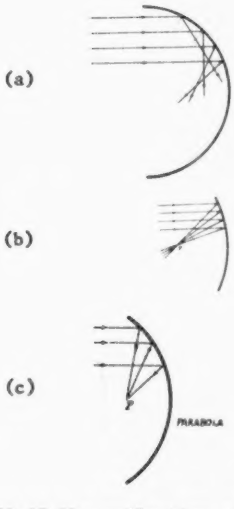
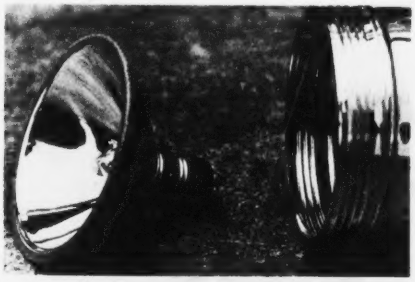


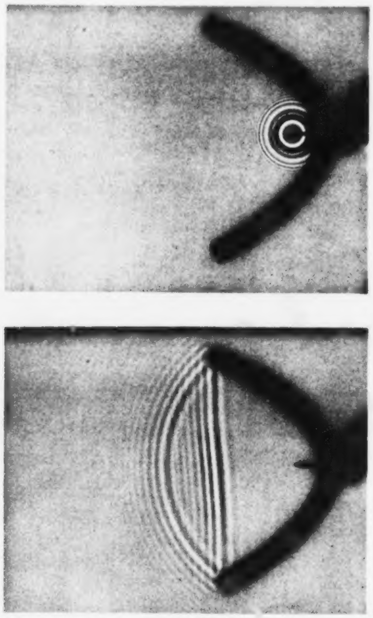
Fig. 12.27 The reflection of parallel wave fronts from circular and parabolic boundaries.

Reflection also occurs from other kinds of surfaces than plane reflectors or mirrors. Many other kinds of wave reflectors are in use today, such as radar antennae and infrared heaters. Figure 12.27 (a) & (b) shows how straight-line waves reflect from two circular reflectors. A few incident and reflected rays are shown. The dotted lines are perpendiculars to the barrier surface; in these cases the perpendiculars are along radius lines of the two circles. While rays reflected from the half circle are headed off in all directions, the rays reflected from the small segment of a circle come close to converging toward a single point. However, a parabolic curve will focus straight-line waves precisely; similarly, a parabolic surface will reflect plane waves to a sharp focus. An impressive example of this is the radio telescope, where huge reflectors are used to detect faint radio waves from space. Conversely, spherical waves originating at the focus will become plane waves on reflection from a parabolic reflector. The automobile headlamp and the flashlight reflector are familiar examples.

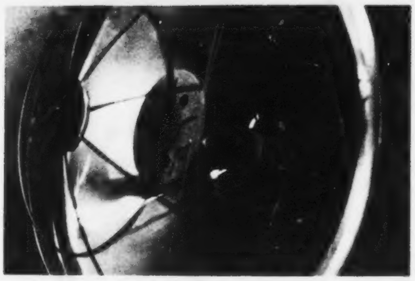
A variety of parabolic reflectors.



Flashlight

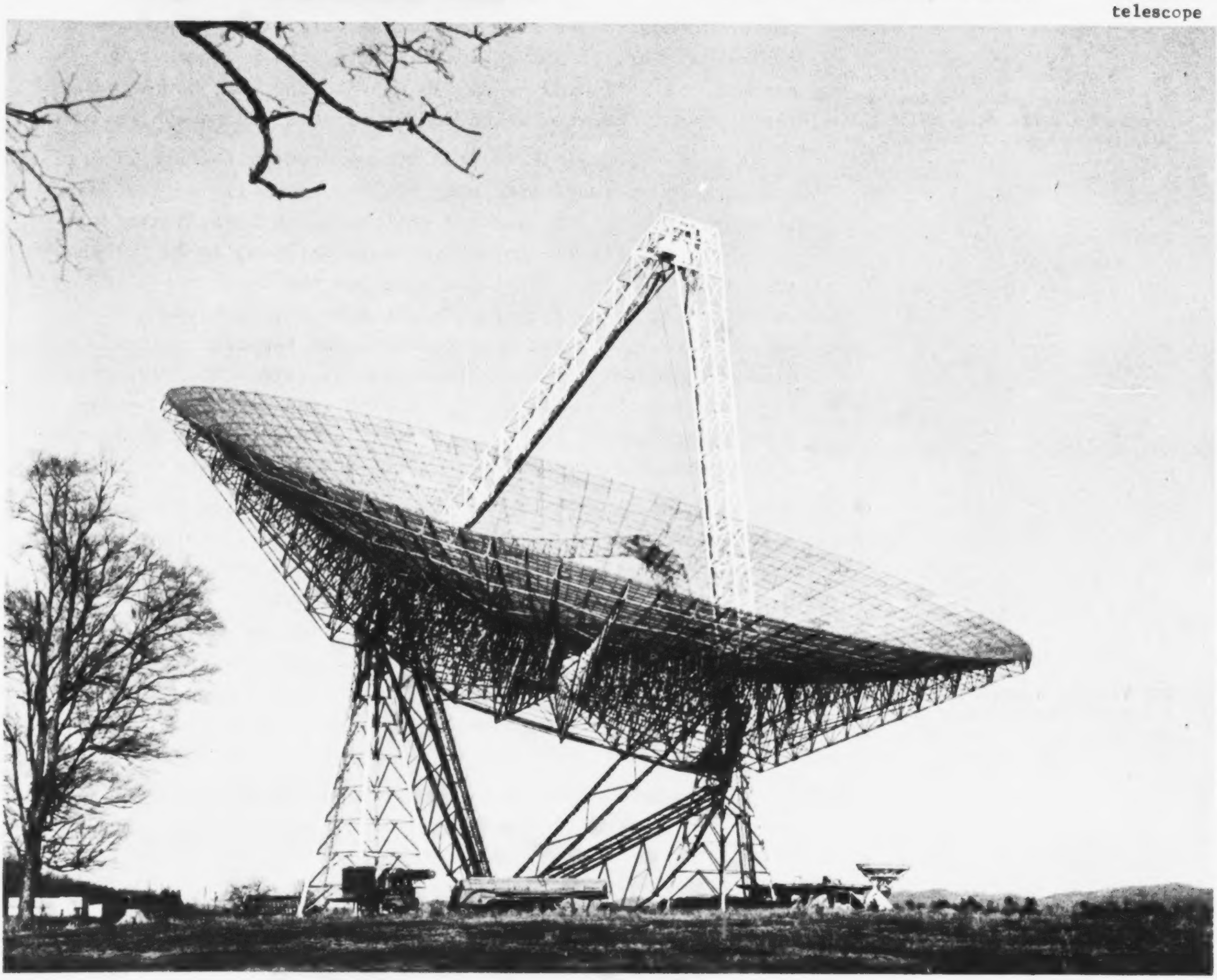


Reflection of a circular pulse in ripple tank.



Radiant heater

Radio telescope



Q26 Why is a "ray" an imaginary line?

Q27 What is the relationship between the angle at which a wave front strikes a barrier and the angle at which it leaves?

Q28 What shape of reflector can converge parallel wave fronts to a sharp focus?

Q29 What happens to wave fronts originating at the focus of such a reflecting surface?

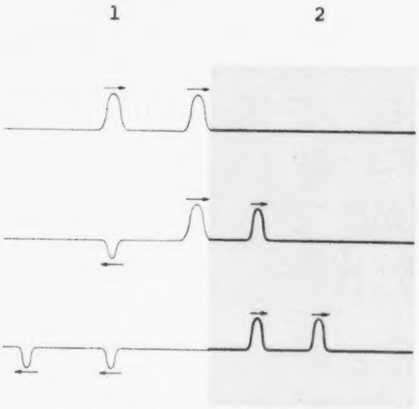


Fig. 12.28. A pair of pulses encountering a medium in which the propagation speed is reduced.

12.10 Refraction. What happens when a wave propagates from one medium to another medium in which its speed of propagation is different? We begin with the simple situation pictured in Fig. 12.28 where two one-dimensional pulses approach a boundary separating two media. We are assuming that the speed of the propagation in medium 1 is greater than it is in medium 2. We might imagine the pulses to be in a light rope (medium 1) tied to a relatively heavy rope or slinky (medium 2). Part of each pulse is reflected at the boundary with the reflected component flipped upside down relative to the incident pulse. (We might have expected this, since the heavier rope is trying to hold the boundary point fixed in a way analogous to the rope reflection at a fixed point discussed earlier.) But we are not particularly interested here in the reflected wave. We want to see what happens to that part of the wave which continues into the second medium.

As shown in Fig. 12.28, the transmitted pulses are closer together in medium 2 than they are in medium 1. Is it clear why that is so? The speed of the pulses is less in the heavier rope, so that the second pulse is catching up during the time it is still in the light rope and the first pulse is already in the heavy rope. In the same way that the two pulses come closer and stay closer, each separate pulse is itself squeezed into a narrower spatial form. That is, while the front of the pulse begins to enter the region of lesser speed, the back of the pulse is still moving with the greater speed, and thus crowding the pulse into a narrower space.

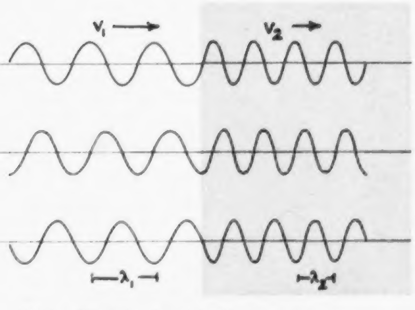


Fig. 12.29. A continuous wave-train entering a medium in which the wave speed is reduced. The effects of the reflected wave train have been neglegted.

Something of the same sort happens to a periodic wave at such a boundary. Figure 12.29 pictures this situation where, for the sake of simplicity, we have assumed that all of the wave is transmitted, and none of it reflected. Just as the two transmitted pulses were brought closer and each pulse was squeezed into a narrower region in Fig. 12.28, so here the spatial pattern is squeezed tighter too. That means that the wavelength λ_2 of the transmitted wave is shorter than the wavelength λ_1 of the incoming, or incident wave.

Although the wavelength changes when the wave passes across the boundary, the frequency of the wave certainly cannot. If the rope is to be unbroken at the boundary, then the pieces immediately adjacent to the boundary and on either side of it must certainly go up and down together. The frequencies of the incident and transmitted waves must, then, be equal. So, since there is no point to labeling them f_1 and f_2, we shall simply label both of them f.

We can write down our wavelength, frequency and speed relationship for both the incident and transmitted waves separately:

$$\lambda_1 f = v_1 \text{ and } \lambda_2 f = v_2 .$$

If we divide one of these by the other, cancelling the f's, we get

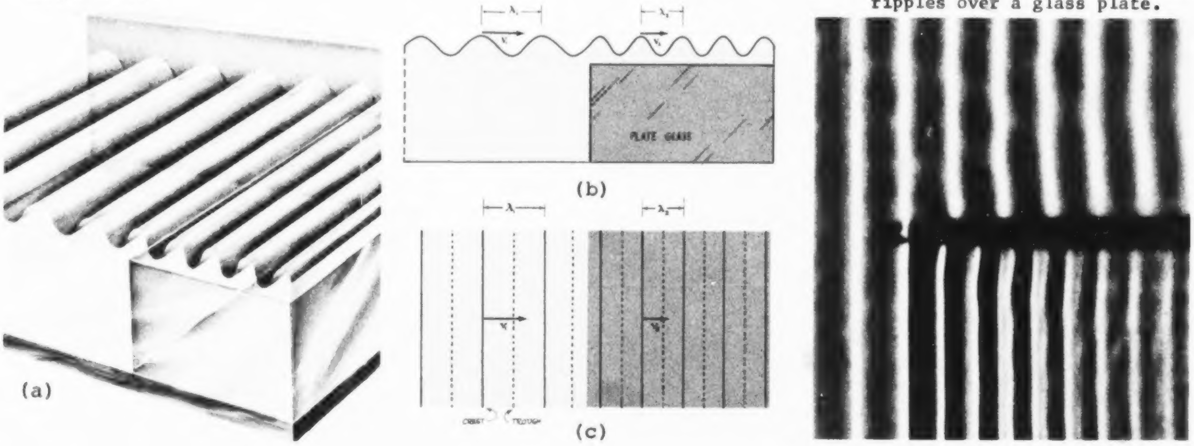
$$\frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} ,$$

which tells that the ratio of the wavelengths in the two media equals the ratio of the speeds.

We can make the same sort of boundary for water ripples. Experiments show that the waves move more slowly in shallower water. A boundary can be created by laying a piece of plate glass on the bottom of a ripple tank to make the water shallower there. Figure 12.30 shows the case where the boundary is parallel to the crest lines of the incident wave.

Figure 12.30(a) is a three dimensional picture of what happens at the boundary. **Figures 12.30**(b) and (c) are cross-section and top views. The photograph Fig. 12.30(d) shows what is actually seen in a ripple tank. The results are the same as those we have already described when a heavy rope is tied to the end of a light one. In fact, the edge view shown in Fig. 12.30(b) could as well be a representation of the rope case.

Fig. 12.30 Refraction of water ripples over a glass plate.



(a)

(b)

(c)

(d)

12.10

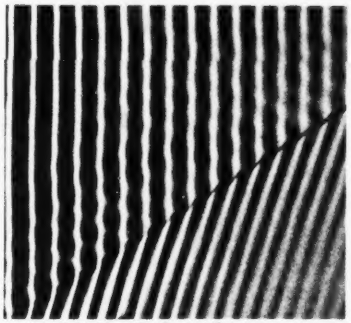


Fig. 12.31 Refraction of water ripples over glass plate.

Water waves offer a possibility not present for rope waves. We can arrange to have the crest lines approach the boundary at an angle. That is, we can see what happens when the boundary is not parallel to the crest lines. The photograph in Fig. 12.31 shows what happens when a ripple tank wave approaches the boundary at the angle of incidence θ_1. Not only do the wavelength and speed change as the wave passes through the boundary, but the direction of the wave propagation changes too. Figure 12.32 illustrates the way this comes about. As each part of a crest line in medium 1 enters medium 2, its speed lessens, and, thus, the crest lines in medium 2 are turned from the orientation they had in medium 1.

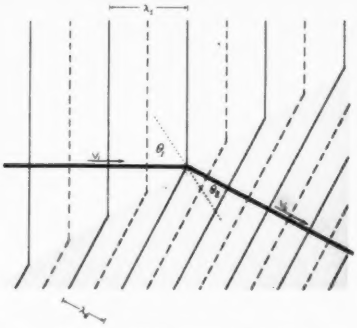
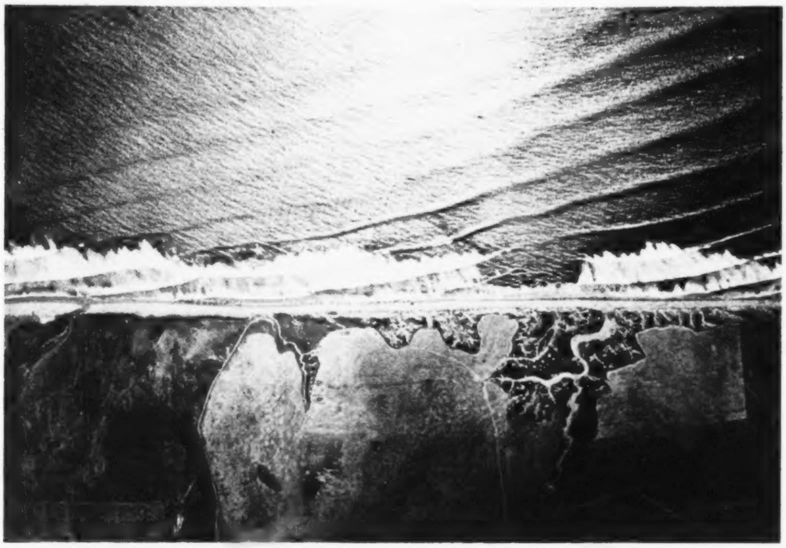


Fig. 12.32. Diagram of rays for refracting wavefronts. Θ_1 and Θ_2 are the angles the rays make with a line (dotted) perpendicular to the boundary.

The ratio Θ_1/Θ_2 is roughly the same as v_2/v_1. The exact relationship is $\sin\Theta_1/\sin\Theta_2 = v_2/v_1$. (See Study Guide 12.3.)

When a wave passes into a medium in which the wave velocity is reduced, the wave fronts are turned so that they are more nearly parallel to the boundary. This is what is pictured in Fig. 12.31, and it accounts for something that you may have noticed if you have been at a beach at an ocean shore. No matter in what direction the waves are moving far from the shore, when they near the beach their crestlines are practically parallel to the shoreline. A wave's speed is steadily being reduced as it moves into water that gets gradually more shallow. So the wave is being refracted continuously as if it were always crossing a boundary between different media, as indeed it is. The refraction of sea waves is so pronounced that wave crests can curl around a small island with an all-beach shoreline and provide surf on all sides.



Refraction of ocean waves approaching shore.

Q30 If a periodic wave slows down on entering a new medium, what happens to (1) its frequency? (2) its wavelength? (3) its direction?

Q31 Sketch roughly what happens to a wave train which enters a new medium where its speed is greater.

12.11

12.11 Sound waves. Sound waves are mechanical disturbances that propagate through an elastic medium, such as the air, as longitudinal or compressional displacements of particles in that medium. Whether the substance of the medium is in a solid, liquid, or gaseous state, longitudinal waves of pressure from some vibrating source will move through the cooperating particles of that material medium. If these waves strike the ear, they can produce the sensation of hearing.

The biology and psychology of hearing is as important to the science of acoustics as is the physics of sound. But for now you may also be interested in the ways in which sound waves exhibit all the properties of wave motion that we have considered thus far in this chapter.

Sound waves can be used to demonstrate all but one of the properties of wave motion in general. The frequencies, velocities, wavelengths and amplitudes of periodic sound waves can be measured, and they can be shown to produce reflection, refraction, diffraction, interference and absorption. Only the property of polarization is missing, because sound waves are longitudinal compression waves.

Vibrating sources for sound waves may be as simple as a tuning fork or as complex as the human larynx with its vocal cords. Sound energy may be transferred through materials as diverse as a quartz crystal and the earth's crust, water of the sea or extremely thin helium gas; in short, any material medium may transmit sound vibrations.

Pulses of sound waves may meet our ears as "clicks" or "dits" or "bams." But when we hear steady tones or buzzes or hums that do not fluctuate, you may be sure that a periodic vibration is generating a periodic wave or set of waves. Audible simple harmonic motion is best illustrated by the steady "pure tone" given off by a tuning fork, although today electronic devices can generate far more "pure" single-frequency sounds than can a tuning fork. The "pitch" of a sound we hear depends on the frequency of the wave.

See "Musical Instruments and Scales" and "Founding a Family of Fiddles" in Project Physics Reader 3.

People can hear waves with frequencies between about 20 cycles per second and 20,000 cycles per second. Dogs can hear over a much wider range (15-50,000 cps), and, as men have only recently learned, bats and porpoises hear and feel frequencies up to about 120,000 cps.

Loudness or volume is a subjective estimate of intensity, and the latter is defined in terms of power, that is, as the am [illegible] perpendicular to a wavefront. So adaptable is the human ear that an exponential scale is used to measure loudness of sound. Figure 12.38 illustrates the wide range of intensities of familiar sounds, relative to a threshold level of 10^{-16} watts per square centimeter.

See "Silence Please" in Project Physics Reader 3.

Relative Intensity	Examples
1	Threshold of hearing (in anechoic chambers)
10^{1}	Normal breathing
10^{2}	(Rustling) leaves in a breeze
10^{3}	Empty movie house
10^{4}	Residential neighborhood at night
10^{5}	Quiet restaurant
10^{6}	Two-person conversation
10^{7}	Busy traffic
10^{8}	Vacuum cleaner
10^{9}	Water at foot of Niagara Falls
10^{10}	Subway train
10^{11}	
10^{12}	Propeller plane at takeoff
10^{13}	Machine-gun fire, close range
10^{14}	Military jet at takeoff
10^{15}	
10^{16}	Wind tunnel (test facility)
10^{17}	Future space rocket (at lift-off)

Fig. 12.38 Levels of noise intensity above 10^{12} times threshold intensity can be felt as a tickling sensation in the ear; beyond 10^{13} times threshold intensity, the sensation changes to pain and may damage the unprotected ear.

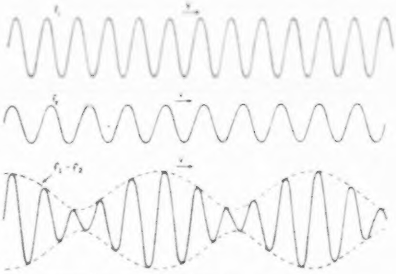
That sound waves take time to travel from source to receiver has always been fairly obvious. Sights and sounds are often closely associated in the same event (lightning and thunder, for instance), and sound is usually late compared to the sight. In 1640 the French mathematician Marin Mersenne first computed the speed of sound in air by timing echoes over a known distance.

It took another seventy years of refinements before William Derham in England, comparing cannon flashes and noises across 12 miles, came close to the modern measurements: sound in air at 68° F moves at 1,125 feet per second.

As for all waves, the speed of sound waves depends on the properties of the medium—the temperature, the density and the elasticity. Sounds travel faster in liquids than in gases, and faster still in solids, because in each state the elasticity of the medium is greater. In sea water, its speed is about 4,800 ft/sec; in steel, it is about 16,000 ft/sec; its fastest known speeds occur in quartz, about 18,000 ft/sec.

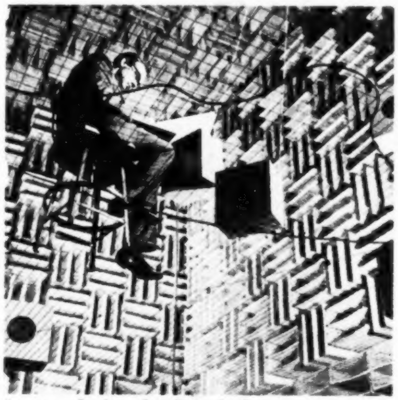
12.11

Interference of sound waves is evident in the acoustically "dead" spots found in many large halls, such as railroad terminals, ballrooms, stadiums and auditoriums. Another interesting example of sound interference is the phenomenon known as beats. When two notes of slightly different frequency are heard simultaneously, they interfere to produce beats, or an intermittent hum. Piano tuners are able to make very fine adjustments in pitch by listening to beats.

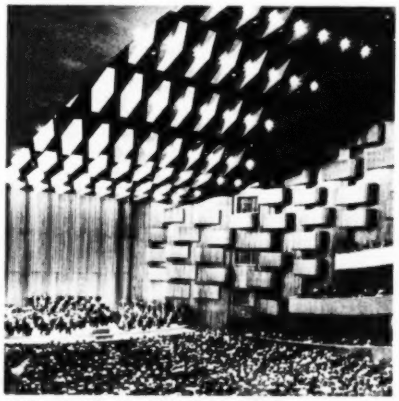


An example of "beats." Two wave trains of different frequencies f_1 and f_2 interfere to produce a wave which fluctuates in amplitude at a frequency ($f_1 - f_2$).

Diffraction is perhaps the most distinctive property of sound waves. Sound waves readily turn around all corners and bend around all barriers to bathe the listener anywhere within range and within the same media. This behavior of sound waves is consistent with Huygens' principle. Any point on a sound wave front meeting an obstacle may act as a new source for a new series of waves radiating in all directions from that point. The ability of sound to diffract through narrow openings or diffract around several sharp corners is surprisingly potent. Sound reflects, like rope or water waves, wherever it encounters a different medium or a boundary to its confinement. Echo chamber effects, often artificially produced by electronics, have become familiar to listeners who enjoy popular music. The architectural accidents called "whispering galleries" (there is one under the dome of the U. S. Capitol) show vividly how sound waves can be reflected to a focus. The weirdly "live" sound of a bare room results from the multiple reflections of waves which are usually absorbed by furniture, rugs and drapes. Scientists have recently devised rooms which maximize echoe called reverberation chambers, and others which minimize them, called anechoic chambers. Both are special laboratories of great value to the study of acoustics.



An anechoic chamber being used for research in acoustics.



Adjustable ceiling surface at the Lincoln Center for the Performing Arts are used to control the acoustical properties of the hall.

Refraction of sound accounts for the fact that we can sometimes see lightning without hearing thunder. It also accounts for most of the shortcomings of sonar devices (sound navigation and ranging) used at sea. Sonic refraction is used for a variety of purposes today, among them the study of the earth's deep structure and seismic prospecting for fossil fuels and minerals.

In 1877 the third Lord Rayleigh wrote The Theory of Sound, which is often considered the culmination of Newtonian mechanics applied to energy transfer. Only a decade earlier the great German physicist-physiologist Hermann von Helmholtz had written The Sensations of Tone, a detailed study of music and hearing. Together these two scientists established the science of sound on a firm Newtonian basis. Acoustics was a marvelous integration of Newtonian science.

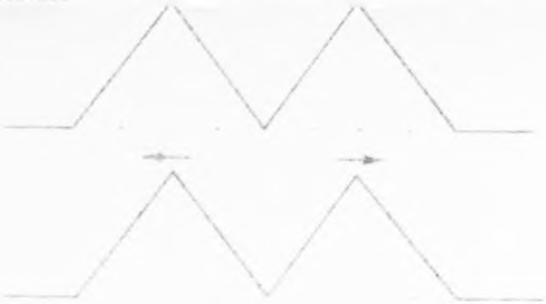
12.11

All this happened before electromagnetic and atomic physics had progressed very far. With the advent of radio waves, of the vacuum tube and electronics, of ultrasonic and infrared research, and of quantum and relativistic physics, the science of acoustics gradually came to seem a very restricted field of study—which might be called "small amplitude fluid mechanics." The wave viewpoint, however, continued to grow in importance.

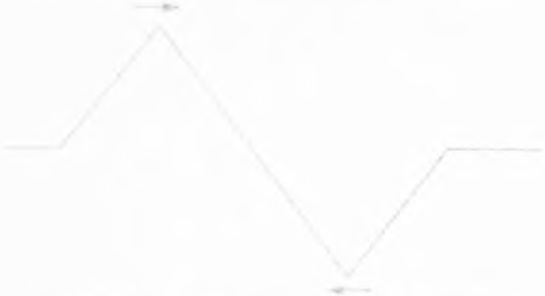
Q32 List five wave behaviors that can be demonstrated with sound waves.

Q33 Why can't sound waves be polarized?

12.1 Pictured are two idealized rope waves at the instants before and after they overlap. Divide the elapsed time into four equal intervals and plot the shape of the rope at the end of each interval.



12.2 Repeat Exercise 12.1 for the two waves pictured below.



12.3 The diagram below shows two successive wave fronts, AB and CD, of a wave train crossing an air-glass boundary.

a) Label an angle equal to angle of incidence θ_A.

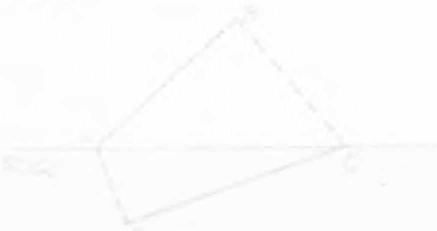
b) Label an angle equal to angle of refraction θ_B.

c) Label the wavelength in air λ_A.

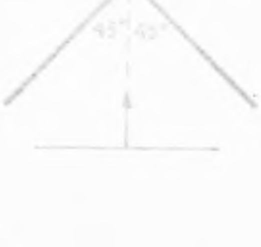
d) Label the wavelength in glass λ_B.

e) Show that $v_A/v_B = \lambda_A/\lambda_B$.

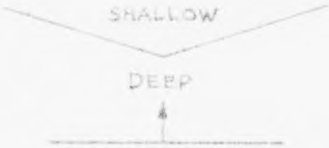
f) If you are familiar with trigonometry, show that $\sin\theta_A/\sin\theta_B = \lambda_A/\lambda_B$.



12.4 A straight-line wave approaches a right-angle reflecting barrier as shown in the figure. Find the shape, size and direction of propagation of the wave after it is completely reflected by the barrier.

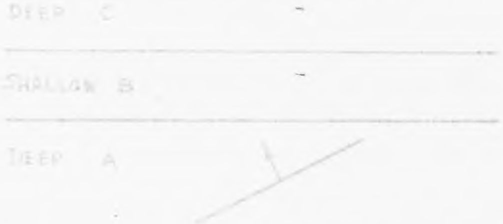


12.5 A straight-line wave in a ripple tank approaches a boundary between deep and shallow water as shown. Describe the shape of the wave as it passes through the boundary and then as it continues in the shallow water.

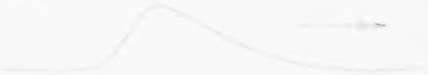


12.6 A periodic ripple-tank wave passes through a straight boundary between deep and shallow water. The angle of refraction is 30°. The propagation speed in the deep water is 0.35 m/sec, and the frequency of the wave is 10 cycles per sec. Find the wavelengths in the deep and shallow water.

12.7 A straight-line ripple-tank wave approaches a narrow region B of shallow water as shown. Prove that the crest line of the wave when in region C is parallel to the crest line shown in region A when regions A and C have the same water depth.



12.8 The wave below propagates to the right along a rope. What is the shape of the wave propagating to the left that could for an instant cancel this one completely?



12.9 The velocity of a portion of rope at some instant is the superposition of the velocities of waves passing through that portion. Is the kinetic energy of a portion of the rope the superposition of the kinetic energies of waves passing through that region?

12.10 Trace the last three curves of Fig. 12.12 and add them graphically to obtain the original curve.

12.11 What kind of interference pattern would you expect to see if the separation between two in-phase sources were less than the wavelength λ? Where would the nodal and and antinodal lines be if the two in-phase sources were separated by the distance λ? By $\lambda/2$? Convince yourself that one additional nodal line appears on each side of the central antinodal line whenever the separation between the two in-phase sources is increased by one wavelength.

12.12 Estimate the wavelength of a 1000 cycles per second sound wave in air; in water; in steel (refer to data in text). Do the same if f = 10,000 cps. Design the dimensions of an experiment to show two-source interference for 1000 cps sound waves.

12.13 If you were to begin to disturb a stretched rubber hose or slinky with a frequency that precisely matched a standing wave frequency, would standing waves appear immediately? If not, what factors would determine the time delay?

12.14 A megaphone directs sound along the megaphone axis if the wavelength of the sound is small compared to the diameter of the opening. Estimate the upper limit of frequences which are diffracted at a cheerleader's megaphone opening. Can you hear what a cheerleader shouts even though you are far off the axis of his or her megaphone?

12.15 Suppose that straight-line ripple waves approach a thin straight barrier which is a few wavelengths long and which is oriented with its length parallel to the wavefronts. What do you predict about the nature of the diffraction pattern along a straight line behind the barrier which is perpendicular to the barrier and passes through the center of the barrier? Why do people who design breakwaters need to concern themselves with diffraction effects?

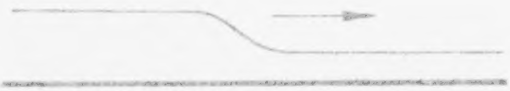
12.16 By actual construction with ruler and compass show that the sketches in Fig. 12.25 appear to be rays originating at S'.

12.17 Sketch the "image" wave for the wave shown in each of the sketches in Fig. 12.26 (a). What relationship exists between the incident image wave and the real reflected wave?

12.18 With ruler and compass reproduced Fig. 12.27 (b) for yourself and find the distance from the circle's center to the point P in terms of the radius of the circle R. Make the radius of your circle much larger than the one in the figure.

12.19 Convince yourself that a parabolic reflector will actually bring parallel wavefronts to a sharp focus. Draw a parabola and some parallel rays along the axis as in Fig. 12.27 (c). Construct line segments perpendicular to the parabola where the rays hit it, and draw the reflected rays at equal angles on the other side of these lines.

12.20 Recalling that water surface waves travel slower in shallow water, what would you expect to happen to the shape of the following wave as it continues to the right? Pay particular attention to the region of varying depth.



Can you use the line of reasoning used above to give at least a partial explanation of the cause of breakers near a beach?

12.21 Look at Fig. 12.32. Convince yourself that if a wave were to approach the boundary between medium 1 and medium 2 from below, along the same direction as the refracted ray in the figure, it would be refracted along the direction of the incident ray in the figure. This is another example of a general rule: if a wave follows a set of rays in one direction, then a wave can follow the same set of rays in the opposite direction. In other words, wave paths are reversible.

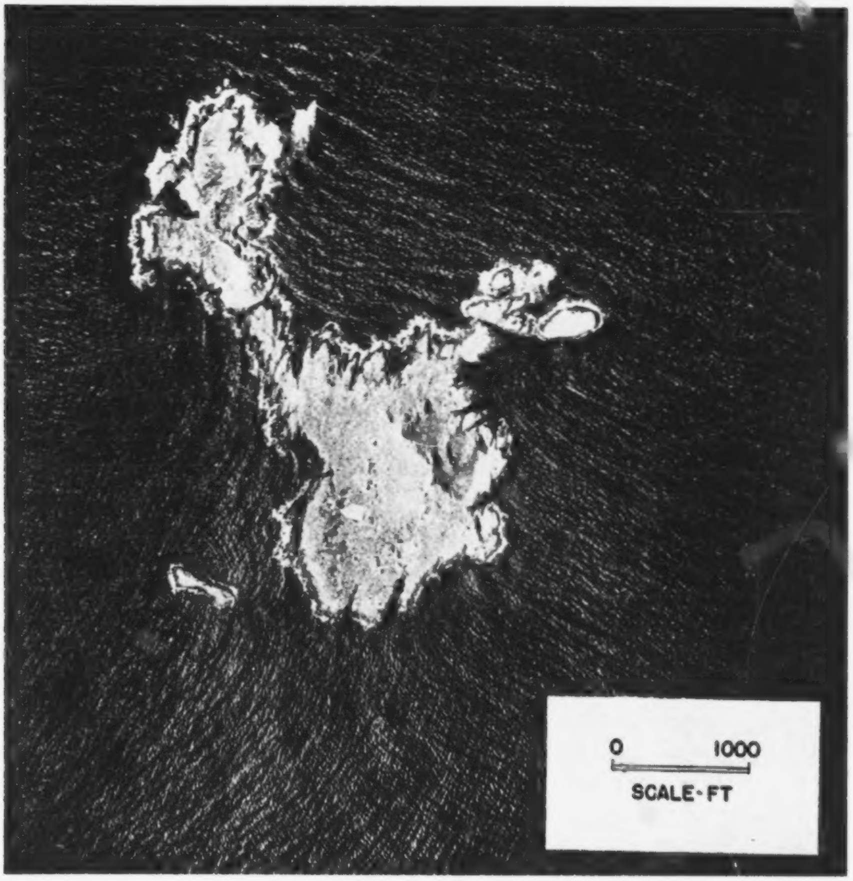
12.22 Can you explain how sound waves are being used to map the floors of oceans?

12.23 When a sound source passes us, whether it be a car horn, a train whistle, or a racing car motor, the pitch we hear goes from high to low. Why is that?

24 Directed reflections of waves from an object occur only when the wavelength is small compared to the dimensions of the object. This is true for sound waves as well as for any other. What does this tell you about the sound frequencies a bat must generate if it is to catch a moth or fly? Actually some bats can detect the presence of a wire about 0.12 mm in diameter. What frequency does that require?

12.25 Suppose you can barely hear in an extremely quiet room a buzzing mosquito at a distance of one meter. What is the sound power output of the mosquito? How many mosquitoes would it take to supply the power for one 100-watt reading lamp? If the swarm were at ten meters' distance, what would the sound be like?

Refraction, reflection, and diffraction of waves around Farallon Island, California. There are breakers all around the coast. The swell coming from top right rounds both sides of the island, producing a crossed pattern below. The small islet 'radiates' the waves away in all directions. (U.S.Navy photograph.)



Epilogue Seventeenth-century scientists thought they would eventually be able to explain all physical phenomena by reducing them to matter and motion. This mechanistic viewpoint later became known as the Newtonian cosmology, since its most impressive success was Newton's theory of planetary motion. Newton and his contemporaries proposed to apply similar methods to other problems (see the Prologue to this unit).

The early enthusiasm for this new approach to science is vividly expressed by Henry Power in his book Experimental Philosophy (1664). Addressing his fellow natural philosophers (or scientists, as we would now call them), he wrote:

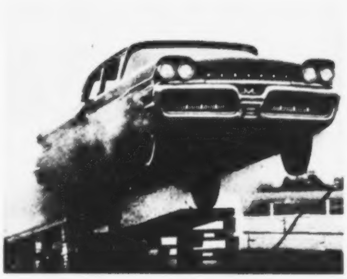
> You are the enlarged and elastical Souls of the world, who, removing all former rubbish, and prejudicial resistances, do make way for the Springy Intellect to flye out into its desired Expansion...
>
> ...This is the Age wherein (me-thinks) Philosophy comes in with a Spring-tide...I see how all the old Rubbish must be thrown away, and carried away with so powerful an Inundation. These are the days that must lay a new Foundation of a more magnificient Philosophy, never to be overthrown: that will Empirically and Sensibly canvass the Phaenomena of Nature, deducing the causes of things from such Originals in Nature, as we observe are producible by Art, and the infallible demonstration of Mechanicks; and certainly, this is the way, and no other, to build a true and permanent Philosophy.

In Power's day there were still many people who did not regard the old Aristotelian cosmology as rubbish to be thrown away. For them, it provided a reassuring sense of unity and interrelationship among natural phenomena which was liable to be lost if everything was reduced simply to atoms moving randomly through space. The poet John Donne, in 1611, lamented the change in cosmology which was already taking place:

And new Philosophy calls all in doubt,
The Element of fire is quite put out;
The Sun is lost, and th'earth, and no man's wit
Can well direct him where to looke for it.
And freely men confesse that this world's spent,
When in the Planets, and the Firmament
They seeke so many new; then see that this
Is crumbled out againe to his Atomies.
'Tis all in peeces, all coherence gone;
All just supply, and all Relation...

Newtonian physics provided powerful methods for analyzing the world, and for uncovering the basic principles that govern the motions of individual pieces of matter. But could it also deal successfully with the richness and complexity of processes that take place in the real world, as well as idealized frictionless processes in a hypothetical vacuum? Can colors, sounds and smells really be reduced to nothing but matter and motion? In the seventeenth century, and even in the eighteenth century, it was too soon to expect Newtonian physics to answer these questions; there was still too much work to be done in establishing the basic principles of mechanics and applying them to astronomical problems. A full-scale attack on the properties of matter and energy had to wait until the nineteenth century.

We have seen in this unit some of the successful generalizations and applications of Newtonian mechanics which were accomplished by the end of the nineteenth century: the conservation laws, new explanations of the properties of heat and gases, and estimates of some properties of molecules. We have introduced the concept of energy to link mechanics to heat, to sound and (in Unit 4) to light, electricity and magnetism. We have also noted that the application of mechanics on a molecular level requires statistical ideas and leads to interesting questions about the direction of time.



Throughout most of the unit we have emphasized the application of mechanics to separate pieces or molecules of matter. But using a molecular model was not the only way to understand the behavior of matter. Without departing from the basic viewpoint of the Newtonian cosmology, scientists could also interpret many phenomena (such as sound and light) in terms of wave motions in continuous matter. By the middle of the nineteenth century it was generally believed that all physical phenomena could be explained by either a particle theory or a wave theory. In the next unit, we will discover how much validity there was in this belief, and we will begin to see the emergence of a new viewpoint in physics, based on the field concept. Then, in Unit 5, particles, waves and fields will come together in the context of twentieth century physics.

Index

Acoustics, 141
Activity, 62
Adams, Henry, 100
Adenosine di-phosphate (ADP), 60, 61
Adenosine tri-phosphate (ATP), 60, 61
Aeolipile, 47
Age of Reason, 4
Air pressure, 91
Airtrack gliders, 13, 17
Amplitude, 141
Anechoic chambers, 143
Angle of incidence, 140
Animal kingdom, 141
Antinodal lines, 125, 126
Arrow of Time, 103
Artisans, 1
Artists, 55
Astronomy, 1
Atoms, 69, 77

Babcock and Wilcox, 56
Bacon, Francis, 9
Barometer, 91
Beats, 143
Bernoulli, Daniel, 81
Billiard ball model, 78
Biology, 60
  of hearing, 141
Burtt, E. A., 6
Blake, 5
Boiler, 57
Boltzmann, 101
Boyle, 2, 77, 81, 91, 97
British Thermal Unit (BTU), 6
Buys-Ballot, 85
Byron, 5

Calorie, 58
Carbohydrates, 60
Carnivores, 60
Carnot, 99
Catalysts, 60
Clausius, 77, 82, 85, 87, 88, 99
Coleridge, 5
Collisions, 13, 15, 17, 18, 24, 26, 79, 85, 87, 93, 100
Computers, 87
Condenser, 51, 52
Conservation laws, 151
  of energy, 7, 42, 43, 92, 95
  of mass 9
  of momentum, 17, 19, 21, 26
  of velocity, 13
Constitution of U.S.A., 5
Cosmology, 151
Coupling, 117
Crests, 119, 140
Cross-section, 87
Curiosity, 47
Cylinder, 49, 51

Darwin, Erasmus, 7
Dead spots, 143
Descartes, 28
Demon, 101
Density, 81, 118
Derham, William, 142
Diffraction, 131, 134, 143
Diffusion, 85
Displacement 120, 144
Distribution, 83, 85
Disturbance, 114
Donne, John, 150
Dry-ice pucks, 13, 17
Duty, 53, 54, 64

Efficiency, 62
Electric fields, 115
Electricity 1, 64, 69
Electromagnetic physics, 144
Electron, 40
Elephant, 91
Energy, 37, 38, 42, 43, 46, 60, 61, 64, 69, 98, 141
Engineering, 47, 64
Entropy, 99, 102, 114
Enzymes, 60
Equal sharing principle, 78
Exploitation of workers, 54

Factory system, 54
Faraday, 64
Fats, 61
Fields, 151
Flammarion, 100
Fluids, 115
Fontenelli, 4
Food, 61
Force, 17, 24, 45
Fourier, 123
Freight cars, 117
Frequency, 119, 120
Frictional processes, 100
Frictionless, process, 151
  table, 24
  wheel, 17
Fuel, 64

Galileo, 91
Gases, 77, 114, 151
Gay-Lussac, 95
Generator, electric, 58
Gibbs, 100
Glucose, 60
God, 2
Goethe, 5, 67
Guericke, 91
Guitar, 42, 129

Harmonics, 123, 128
Heat, 1, 11, 46, 58, 59, 62, 77, 97, 114, 151
Heaven, 2
Helmholtz, 68, 143
Herapath, 82
Herbivores, 60
Heron, 47
Horsepower, 53
Human body, 61
Huygens, 25, 27, 132, 143

Ideal gas law, 96
Image, 135
Impact, 25
Industrial Revolution, 54
Intellectuals, 55

Interference, 124, 125, 143
Inventors, 47
Irreversibility, 78, 98, 100

Jerusalem, 55
Johnson, Samuel, 5
Joule, James, 58, 66, 77, 82
Joule, unit, 53

Kelvin, 90, 100
Kilocalorie, 58, 61
Kinetic energy, 37, 40, 77, 95
Kinetic theory, 103

Laplace, 7
Lavoisier, 11
Laws of motion, 3
Leibniz, 27, 43, 59
Life, 60
Light, 1, 11
Line of cars, 117
Locomotive, 117
Locke, John, 4
Loschmidt, 89, 90
Loudness, 142
Lucretius, 7, 9

Mack, 103
Magnetism, 1, 64
Magnets, 42
Mass, 9, 11, 118
Mass production, 54
Mathematics, 129
Matter, 9
Maxwell, 83, 88, 89, 102
Mayer, 66, 68
Mean free path, 87
Mechanical energy, 77
Medium, 114, 136, 141
Mersenne, Marin, 142
Mitochondria, 60, 61
Model, 77, 79, 151
Moderation, 5
Molecules, 77
Momentum, 17, 19, 21, 92, 95
Money, 47
Motion, 13

Nature-philosophy, 6, 55, 67
Neutrino, 69
Newcomin, 48, 49, 50, 52
Newton, 1, 4, 9
Newton's laws, 19, 43, 47, 93, 113
Newtonian mechanics, 5, 77, 151
Nodal lines, 125, 126
Nuclear reactor, 70
Nucleus, 40

Oblique reflection, 135
Omnivores, 60
Organ pipes, 129
Overtones, 129

Paddle-wheel, 58
Pair of Sources, 133
Parabolic curve, 136
Paradox, 103
Parsons, 56
Particles, 1, 113
Pascal, 91
Pauli, 69
Pendulum, 119
Period, 118, 120
Philosophers, 4, 13, 28
Philosophy, 1, 66
Photosynthesis, 60
Physicist, 19, 37
Piston, 49, 92
Plane, 116
Planets, 1
Plants, 60
Poincare, 70
Polarization, 116
Politics, 5
Pope, Alexander, 5
Population, 60, 62, 63
Potential energy, 40, 42, 45, 58
Power, 53, 54
Power, Henry, 91, 150
Pressure, 48, 52, 81, 91, 92, 94, 114, 115
Principia, 3, 22, 81
Profits, 54
Propagate, 116
Proteins, 61
Psychology, 141
Pulses, 117, 118, 119, 125, 138
Pure tone, 141
Pythagorean ideal, 129

Quality, 129
Quantum, 78, 144

Radio waves, 144
Railway crossing, 117
Randomness, 78, 80
Range of frequencies, 141
Rayleigh, 143
Rays, 136
Reactions, checmial, 9
Recoil of cannon, 22
Recurrence paradox, 103
Reflected wave, 128
Reflection, 134, 136
Reflector, 136
Refraction, 138, 143
Relativistic physics, 114
Restoring force, 118
Reverberation chambers, 143
Reversibility paradox, 103
Ripples, 121
Ripple tank, 139
Rocket, 22
Rogers, 67
Romantic movement, 5, 55
Rope, 114, 116, 128
Royal Society, 24

Satanic mills, 55
Savery, 48
Saxophone, 129
Schelling, 6, 66
Scientists, 46, 150
Scott, 5
Seebeck, 64
Shelley, 5
Shoreline, 140
Simple harmonic motion, 119
Sizes of molecules, 85

Slit width, 133
Society, 4
Solar system, 1
Sound, 133, 141, 151
Source, 114
Speed, 43, 81, 82, 93, 94, 118, 139, 142
Spike heels, 91
Spring, 115, 116
Standing wave, 128
Starvation, 55, 62
Stationary waves, 128
Statistical law, 83
Steam engine, 47, 98, 99
Stern, Otto, 85
Stiffness factor, 118
Stokes, 88
Straight barrier, 135
Sunlight, 60
Superposition, 123, 128, 129
Surf, 140
Swift, Jonathan, 5
Synthesis, 60
System, 18, 19, 23, 24, 28

Target, 83
Temperature, 62, 81, 83, 88, 95, 96, 97, 114
Thermodynamics, 69, 98, 99
Thermostat, 97
Threshold of sound, 142
Time's Arrow, 103
Torricelli, 91
Torsional wave, 115
Townely, 91
Transformation of energy, 43
Transmitted wave, 138
Trough, 119
Trumpet, 129
Turbine, 56
Two-source interference, 124

Ultrasonic frequencies, 141

Vector sum, 13, 18
Velocity, 13, 83, 142
Violin, 129
Viscosity, 88
Vis viva, 27, 28, 43, 59
Volta, 64

Wallis, 25
Water, pumping of, 48, 53
Watt, James, 51
Watt, unit, 53
Wave, 113 ff, 128, 135, 151
Wave front, 131
Wavelength, 119, 120, 126
Wave motion, 144
Wave propagation, 117
Wells, H. G., 100
Western civilization, 54
Wheels, frictionless, 17
Whispering galleries, 143
Wilson, James, 5
Wordsworth, 5
Work, 37, 40, 44, 64
World machine, 1
Wren, Christopher, 25

Zartmann, 85
Zermelo, 103

Brief Answers to Unit 3 Study Guide

Chapter 9

9.1 Discussion
9.2 Yes
9.3 No
9.4 Discussion
9.5 (a) 220.2g
(b) 20.2g
9.6 (a) Discussion
(b) Yes
(c) Yes
9.7 0.8 m/sec
9.8 Derivation
9.9 3.33×10^{-6} kg
9.10 Discussion
9.11 Discussion
9.12 Discussion
9.13 1200 kg-m/sec
400 nts
30 m
9.14 (a) 200 kg-m/sec
(b) 10 nts
9.15 Yes
9.16 Derivation
9.17 (a) 0
(b) 0
$M_c\vec{V}_c + M_s\vec{V}_s$
(c) $M_c\vec{V}_c = -M_s\vec{V}_s$
(d) $V_s/V_c = M_c/M_s$
(e) 10 m/sec
9.18 Derivation
9.19 Left car
(1) speed of one car or
(2) mass of cars
Distance
Force
9.20 Discussion
9.21 Derivation
9.22 One-half initial speed in opposite directions
9.23 (a) No
(b) Continues forward
(c) Continues backward
9.24 .8 kg-m/sec forward
.8 kg-m/sec away
1.6 kg-m/sec away
No
9.25 4.2 kg-m/sec, 1260 kg-m^2/sec^2
0.171 kg, 427.5 kg-m^2/sec^2
0.075 kg-m/sec, 0.1125 kg-m^2/sec^2
111.0 m/sec, 1.62×10^4 kg-m/sec
10^{-4} kg, 0.2 m/sec
9.26 Discussion
9.27 No, discussion

Chapter 10

10.1 1.82×10^{-18} j
5.5×10^{17} electrons
10.2 (a) 67.5 j
(b) 4.5×10^9 j
(c) 3750 j
(d) 2.7×10^{33} j
10.3 Discussion
10.4 (a) .00225 j
(b) .056 j
10.5 Discussion
10.6 (a) $5 m/sec^2$
19 sec
95 m/sec
(b) 95 m/sec
10.7 Derivation
10.8 22°
10.9 9.3 min
10.10 Derivation
10.11 (a) 9.6×10^9 j
(b) 4.8×10^5 nt
(c) 7.68×10^6 watts
(d) 880 m
(e) 1 & 2 increase
3 decreases
(f) change direction
10.12 (a) 1046 times
(b) increase weight
decrease amount of water
increase height of fall
lower specific heat
10.13 0.12°C
10.14 Discussion
10.15 3 weeks
10.16 1.5 kg
10.17 Discussion
10.18 Discussion
10.19 Discussion

10.20 Undamaged rocket
10.21 1800 nts
90 j
10.22 Discussion
10.23 Discussion

Chapter 11

11.1 Pressure, density, temp., viscosity
11.2 Discussion
11.3 Discussion
11.4 Working model, Theoretical model
11.5 Discussion
11.6 No, discussion
11.7 (a) probable speed
(b) average speed
(c) no negative speeds
11.8 $P \quad T$
11.9 (a) 10^{-9}m
(b) 10^{-9}m
11.10 Discussion
11.11 Discussion
11.12 Density changes
11.13 50 atmospheres
11.14 5 atmospheres
11.15 Discussion
11.16 Discussion
11.17 Discussion
11.18 Discussion
11.19 Discussion
11.20 Discussion
11.21 Discussion

Chapter 12

12.1 Construction
12.2 Construction
12.3 (a) BAC = θ_A
(b) ACD = θ_B
(c) BC = λ_A
(d) AD = λ_B
(e) Derivation
(f) Derivation
12.4 Straight-line
12.5 Two straight-line waves inclined toward each other
12.6 $\lambda_D = .035$ m
$\lambda_s = .025$ m
12.7 Derivation
12.8 Reflection
12.9 No
12.10 Construction
12.11 Construction
12.12 Discussion
12.13 No, discussion
12.14 100 and 1000 cps
yes
12.15 Maximum
12.16 Construction
12.17 Construction
12.18 R/2
12.19 Construction
12.20 Discussion
12.21 Discussion
12.22 2d = vt
12.23 Doppler Effect
12.24 3×10^4 cps
2.5×10^6 cps
12.25 1.27×10^{-11} watts
8×10^{12} mosquitoes
wind-tunnel test

Picture Credits

Cover photograph: "Locomotive Wheels" by Farrell Grehan.

Prologue

Pp. 0, 3 Albert B. Gregory, Jr.

P. 1 Swiss Federation of Watch Manufacturers.

P. 2 Whitworth Art Gallery, University of Manchester.

Chapter Nine

P. 8 Pictorial Parade, N.Y.C.

P. 9 Dr. Harold E. Edgerton, Massachusetts Institute of Technology

P. 10 (thunderhead) Peter M. Saunders, Woods Hole Oceanographic Institute; (bonfire) Colgate University.

P. 12 (Lavoisier portrait) painted by Jacques Louis David. Courtesy of The Rockefeller University.

P. 13 Boston Patriots Football Club.

P. 18 (auto collision) The Bettmann Archive; (hockey players) Pictorial Parade, N.Y.C; (space vehicle) National Space & Aeronautics Administration, (galaxy) Mount Wilson and Palomar Observatories.

P. 19 National Film Board of Canada.

P. 20 Emil Schulthess. Black Star Publishing Company, Inc.

P. 22 The Boeing Company.

P. 23 Mount Wilson and Palomar Observatories.

P. 24 Henry Groskwsky, LIFE MAGAZINE, © Time Inc.

P. 28 (Leibniz) Burndy Library, Norwalk, Conn. (Descartes) the Franz Hals portrait in the National Art Museum, Copenhagen.

P. 29 (Huygens) Royal Netherlands Academy of Sciences and Letters, Amsterdam; (Wren) painting by Sir Godfrey Kneller, 1711, National Portrait Gallery, London; (St. Paul's Cathedral) photo by Helmut Gernsheim from his book Beautiful London.

P. 35 Dr. Harold E. Edgerton, M.I.T.

Chapter Ten

P. 36 A. Schatz, LIFE MAGAZINE, © Time Inc.

P. 37 H. Grossman, LIFE MAGAZINE, © Time Inc.

P. 39 Emil Schulthess. Black Star Publishing Company, Inc.

P. 41 Lee Equipment Company, Boston.

P. 43 (guitar string) Albert B. Gregory, Jr., (golf ball) Dr. Harold E. Edgerton, M.I.T.

P. 44 Dimitri Kessel, LIFE MAGAZINE, © Time Inc.

P. 46 (camel & waterwheel) C.W. Kirkland, LIFE MAGAZINE, © Time Inc, (boy on waterwheel) LIFE MAGAZINE, © Time Inc, (reversible overshot waterwheel) Agricola, Georgius, De Re Metallica, 1556, Houghton Library, Harvard University, (windmill) M.G. Walker, LIFE MAGAZINE, © Time Inc.

P. 47 (sprinkler) Dr. Harold E. Edgerton. M.I.T.; (aeolipile) Science Museum, London.

P. 50 Newcomen Society, London.

P. 51 (Watt in his laboratory) Figuier, Louis, Les Merveilles de la Science, Furne, Jouvet et Cie, Paris; (portrait) engraved by S.W. Reynolds after the oil portrait by C.F. von Breda, 1793, Science Museum, London.

P. 54 ("Rocket") Figuier, Louis, Les Merveilles de la Science, Furne, Jouvet et Cie, Paris; Science Museum, London.

P. 55 (train) Boston & Maine Corporation; (steamboat) Science Museum, London.

Pp. 56, 57 (blacksmith) W. Fitzenthaler, Photography Collection, Carpenter Center for the Visual Arts, Harvard University; (steel workers) Jones & Laughlin Steel Corp., Pittsburgh; (steam turbine rotor) Franco Tosi, S.p.A. Legnano.

P. 58 American Institute of Physics.

P. 60 Professor Keith R. Porter, Dept. of Biology, Harvard University.

P. 63 "The Repast of the Lion" by Henri Rousseau, courtesy of The Metropolitan Museum of Art, bequest of Samuel A. Lewisohn, 1951.

P. 66 American Institute of Physics.

P. 67 (Goethe) painting by Angelica Kauffmann, 1787; (Schelling) pastel by Friedrich Tieck, 1801, collection of Dr. Hermann von Schelling.

P. 68 Koenigsberger, Leo, Hermann von Helmholtz, Braunschweig Druck und Verlag, von Friedrich Vieweg und Sohn.

P. 71 Ron MacNeil, Boston.

P. 73 from "The Last Great Cruise" advertisement by Kenyon & Eckhardt, Inc., for the Diners Club Inc. and Fugazy Travel Bureau, Inc., General Sales Agents for the City of Long Beach, California.

Chapter Eleven

P. 76 Courtesy AMF-Voit.

P. 79 (balloon) U.S. Air Force.

P. 81 Burndy Library, Norwalk, Conn.

P. 82 Courtesy of Professor Eric Mendoza.

P. 104 (1) Courtesy of Trinity College Library, Cambridge; (2) Burndy Library, (3) Thompson, Silvanus P., Life of William Thomson, vol. 1, MacMillan and Co., Ltd., London, (4) American Institute of Physics, (5) Deutsches Museum, Munich.

P. 106 (colliding balls) PSSC Physics, D.C. Heath & Co., 1965; (light bulb) Dr. Harold E. Edgerton, M.I.T., (bonfire) Colgate University; (landscape) "Mount Williamson—Clearing Storm," 1944, Ansel Adams.

P. 107 P.E. Genereux, E. Lynn.

P. 109 Greek National Tourist Office, N.Y.C.

Chapter Twelve

P. 112 Magnum Photos Inc., N.Y.C. Werner Bischof.

P. 117 Union Pacific Railroad.

P. 122 (portrait) Oeuvres de Fourier, vol. 2, Gauthier et Fils, Paris, 1890.

Figs. 12.13, 12.14, 12.30(d), 12.31, and pp. 124, 131, 133, 135, (ripple tank photos) p. 137. Courtesy, Film Studio Educational Development Center.

P. 128 "Singer with Lyre," from red-figured amphora attributed to the Brygos painter. Courtesy of Museum of Fine Arts, Boston. John Michael Rodocanachi Fund.

P. 130 National Film Board of Canada.

P. 132 Courtesy, College of Engineering, University of California, Berkeley.

P. 137 (radiotelescope) National Radio Astronomy Observatory, Green Bank, W. Va.

P. 140 U.S. Navy

P. 143 (anechoic chamber) Bell Telephone Laboratories; (Philharmonic Hall) c Lincoln Center for the Performing Arts, Inc., 1967—photo by Bob Serating.

P. 145 George Silk, LIFE MAGAZINE, © Time Inc.

P. 149 U.S. Navy.

All photographs not credited above were made by the staff of Harvard Project Physics.

Acknowledgments

Prologue

P. 1 Desagulier, J.T., The Newtonian System of the World, the Best Model of Government, an Allegorical Poem.

PP. 1-2 Descartes, René, The Philosophical Works of Descartes, trans. Haldane, Elizabeth S. and Ross, G.R.T., Cambridge University Press, Vol. 1, p. 299.

P. 2 "A Free Inquiry into the received Notion of Nature," The Works of Robert Boyle, Vol. IV, p. 362.

P. 3 Newton, Sir Isaac, The Principia, Motte's translation revised by Florian Cajori, University of California Press, preface to first edition (1687), p. xviii.

P. 4 Fontenelle, "Preface sur L'Utilité des Mathématiques et de la Physique" from Oeuvres Complètes (1818), in Randall, John H., Jr., The Making of the Modern Mind, Houghton Mifflin Company, p. 254.

P. 5 Wilson, James, The Works of the Honourable James Wilson, published under direction of Wilson, Bird, Bronson and Chauncey, pp. 411-12, not inclusive.

P. 6 Burtt, E.A., The Metaphysical Foundations of Modern Science, Humanities Press Inc., pp. 238-39.

P. 7 Laplace, A Philosophical Essay on Probabilities, first published in Paris (1814), trans. Truscott, Frederick and Emory, Frederick, Dover Publications, Inc., p. 4.

P. 7 Darwin, Erasmus, quoted by Charles Darwin in his preface to Krause, Erasmus Darwin, D. Appleton and Company, p. 44.

Chapter Nine

P. 9 Lucretius, On the Nature of Things, trans. H. Munro, George Bell & Sons, Book II, p. 51.

P. 9 Bacon, Francis, Novum Organum.

P. 11 Lavoisier, Antoine, Elements of Chemistry, trans. Robert Kerr, Dover Publications, Inc., p. 130.

P. 27 Gerhardt, Die Philosophische Schriften von Gottfried Wilhelm Leibniz, 1875-90, Vol. IV, p. 286, Eng. trans. in Dugas, R., Mechanics in the Seventeenth Century, Central Book Co., p. 407.

P. 27 Ibid.

P. 27 Leibniz, Gottfried Wilhelm, "Essay de Dynamique sur les loix du Mouvement" (1691), first published in Gerhardt, op. cit., Vol. VI, pp. 215-31, Eng. trans. in Dugas, R., op. cit., p. 475.

P. 27 Descartes, René, Principia Philosophiae (1644), Vol. II, p. 36, in Mach, Ernst, The Science of Mechanics, Open Court Publishing Company, sixth edition, p. 361.

Chapter Ten

P. 49 Darwin, Erasmus, Botanic Garden - The Economy of Vegetation, J. Moore, p. 49.

P. 50 Savery. Thomas, in Hart, I.B., James Watt and the History of Steam Power, Abelard-Schuman, p. 81.

P. 56 Babcock and Wilcox Company, The Story of Steam.

P. 59 Joule, James Prescott, The Scientific Papers of James Prescott Joule, Taylor and Francis, p. 328.

P. 66 Ibid., p. 158.

P. 68 Helmholtz, H., Popular Lectures on Scientific Subjects, trans. Atkinson, E., D. Appleton and Company, p. 170.

P. 70 Poincaré, Henri, La science et l'hypothese, E. Flammarion, excerpt trans. by McClain, John.

Chapter Eleven

P. 88 Memoir and Scientific Correspondence of the Late Sir George Gabriel Stokes, bart., ed. Larmer, Sir J., Cambridge University Press, Vol. II, pp. 8-11, not inclusive.

P. 89 Campbell, Lewis and Garnett William, The Life of James Clerk Maxwell, Macmillan, p. 318.

P. 90 Lord Kelvin, "The Size of Atoms, "Nature, Vol. I, p. 551

p. 95 Maxwell, James Clerk, Cambridge University unpublished notes, published by permission of Brigadier Wedderburn-Maxwell.

P. 98 Frost, Robert, "West-Running Brook," Complete Poems of Robert Frost, Holt, Rinehart and Winston, Inc., p. 33.

P. 106 Lord Kelvin, "The Kinetic Theory of the Dissipation of Energy," Proceedings of the Royal Society of Edinburgh, Vol. 8, p. 325.

P. 107 Nietzsche, Friedrich, "Der Wille zur Macht," Nietzsche: An Anthology of His Works, ed. Manthey-Zorn, Otto, Washington Square Press, p. 90.

P. 107 Poincaré, Henri, Kinetic Theory, ed. Stephen Brush, Pergamon Press, Vol. 2, p. 206.

P. 107 Shelley, Percy Bysshe, Hellas, ed. Wise, Thomas J., Reeves & Turner, pp. 51-52, not inclusive.

Chapter Twelve

P. 132 Huygens, Christiaan, Treatise on Light, first published in Paris (1690), trans. Thompson, Silvanus, Macmillan and Company, p. 19.

Epilogue

P. 150 Power, Henry, Experimental Philosophy, p. 192.

P. 150 Donne, John, "The First Anniversary," Donne's Poetical Works, ed. Grierson, Herbert J., Oxford University Press (Clarendon Press imprint), Vol. 1, p. 237.

Answers to End of Section Questions

Chapter 9

Q1 No. Don't confuse mass with volume.

Q2 a) No; its mass increases by 2000 tons a year.
b) The solar system.
c) The earth is very nearly a closed system; 2,000 tons is a very small fraction of the earth's mass.

Q3 Answer (c).

Q4 All three quantities are not in general conserved; it is the product of mass and velocity which is conserved.

Q5 In cases (a), (c) and (d) the two carts will stop after the collision, since their momenta before collision are equal in magnitude and opposite in direction.

Q6 Least momentum: a pitched baseball (very small mass and fairly small speed).
Greatest momentum: a jet plane in flight (very large mass and high speed).

Q7 a) about 4 cm/sec. Faster ball delivers more momentum to girl.
b) about 4 cm/sec. More massive ball delivers more momentum to girl.
c) about 1 cm/sec. With some gain in momentum, more massive girl gains less speed.
d) about 4 cm/sec. Momentum change of ball (a vector!) is greater if its direction reverses.
(These answers assume the mass of the ball is much less than the mass of the girl.)

Q8 a) $\Delta(mv) = F(\Delta t) = (35 \times 10^{6} \text{ newtons}) \times (150 \text{ sec.}) = 5.2 \times 10^{9} \text{ kg m/sec.}$
b) Mass of rocket changes as it burns fuel.

Q9 In cases (a), (b) and (d), Δt is lengthened, thereby decreasing F. In case (c), Δt is short, making F large.

Q10 No, not if the girl and skateboard are an isolated system. (In fact, a skillful skateboarder can use frictional forces to make sharp turns.)

Q11 Answer (c).

Q12 In cases (a) and (b) the earth exerts a net force on the system. In case (c) the sun exerts a net force on the system.

Q13 Answer (c); vis viva is conserved only if the colliding objects are very hard.

Q14 False. Huygens explained the demonstration.

Q15 With parallel strings, the balls will collide horizontally.

Q16 Answer (c); the square of any number is positive.

Q17 Answer (c).

Q18 He said that the vis viva was "dissipated among the small parts" of the colliding bodies.

Chapter 10

Q1 Answer (a).

Q2 Answer (e).

Q3 about .2 meters.

Q4 Answer (c).

Q5 Answer (c). An increase in potential energy equals the work you do on the spring.

Q6 Answer (e). You must do work on the objects to push them closer together.

Q7 Answer (e). Kinetic energy increases and gravitational potential energy decreases. Their sum stays the same (if air resistance is negligible).

Q8 Kinetic energy is greatest at midpoint, where string is unstretched. Potential energy is greatest at extreme position, where the speed of the string is zero.

Q9 Both will gain the same amount of kinetic energy (conservation of mechanical energy). The less massive treble string will gain more speed, however.

Q10 No. The force on Tarzan is inward along the radius while he moves along a circular arc.

Q11 Answer (c).

Q12 Some increase in gravitational potential energy in both cases.

Q13 Answer (c).

Q14 False. It was the other way around.

Q15 Savery's patent prevented Newcomen from profiting much from his engine.

Q16 Answer (c).

Q17 Answer (c).

Q18 Answer (e).

Q19 Answer (b).

Q20 Nearly all. A small amount was transformed in kinetic energy of the slowly descending weights.

Q21 Answer (a).

Q22 Answer (e).

Q23 Answer (c).

Q24 Answer (a).

Q25 Answer (d).

Q26 Answer (c).

Q27 Joule's approach was experimental, whereas Mayer's was theoretical (although he used data which had resulted from the experiments of others).

Chapter 11

Q1 Answer (c).

Q2 It is assumed that Newton's laws do apply.

Q3 False. Quantum mechanics is needed to treat the motion of atoms within molecules.

Q4 Answer (b).

Q5 In gases the molecules are far enough apart that the rather complicated intermolecular forces can safely be neglected.

Q6 Answer (b).

Q7 Answer (c).

Q8 Answer (d).

Q9 Answer (c).

Q10 Answer (c). Greater speed means greater momentum change in each collision and also more collisions per second.

Q11 Molecules will bounce off the piston with greater speed than before, so the total kinetic energy will increase.

Q12 Answer (a) is the ideal gas law and answer (b) is the prediction of kinetic theory. When combined they result in the prediction that $\overline{KE}$ is proportional to T.

Q13 Answers (a), (b) and (c) are consistent with the second law of thermodynamics.

Q14 Answer (a).

Q15
a) An unbroken egg is more ordered.
b) A glass of ice and warm water is more ordered.

Q16
a) True
b) False
c) False
Maxwell's demon is an imaginary, hypothetical device.

Q17 Answer (b).

Q18 Answer (c).

Q19 Answer (a). The ancient philosophers postulated recurrence but did not "discover" it as a theorem in mechanics.

Chapter 12

Q1 Transverse, longitudinal, and torsional waves.

Q2 Only longitudinal waves. Fluids can be compressed, but they are not stiff enough to be bent or twisted.

Q3 Transverse waves.

Q4 No. The movement of the bump in the rug depends on the movement of the mouse; it does not go on by itself.

Q5 Energy. (Particles of the medium are not transferred along the direction of wave motion.)

Q6 Stiffness and density.

Q7
1) Wavelength, amplitude, polarization.
2) Frequency, period.

Q8 Wavelength is the distance between two points of the wave that are moving in the same way. The wave does not have to be a sine wave; any repeating wave pattern has a wavelength.

Q9
1) 100 cycles per second.
2) 0.01 second.
3) $\text{wavelength} = \dfrac{\text{speed}}{\text{frequency}} = \dfrac{10\text{ m/sec}}{100\text{ cycles/sec}}$
$= 0.1$ meter (per cycle).

Q10 Answer (b).

Q11 Greatest displacement of point P is $(A_1 + A_2)$.

Q12 Wave amplitudes are positive or negative quantities; they add algebraically.

Q13 Nodal lines are regions of cancellation.

Q14 Antinodal lines are regions of reinforcement; the amplitude there is greatest.

Q15 Answer (a)

Q16 Waves from two in-phase sources arrive at a point out of phase if the point is one-half wavelength (or 3/2, 5/2, 7/2, etc.) farther from one source than the other.

Q17
1) No motion at the nodes.
2) Greatest motion at the antinodes.

Q18 Distance between nodes is $\lambda/2$.

Q19 Wavelength = 2L, so that one-half wavelength just fits on the string.

Q20 No. The frequency must be one for which the corresponding wavelength is such that 1 or 2 or 3 or ... half-wavelengths fit between the boundaries of the medium.

Q21 All points on a wave front have the same phase; that is, they are in the same state of motion.

Q22 Every point on a wave front may be considered to behave as a point source for waves generated in the direction of the wave's propagation.

Q23 At no point in space is the distance to one edge of the slit one-half wavelength farther than the distance to the other edge.

Q24 As the wavelength increases, the diffraction pattern becomes more spread out and the number of nodal lines decreases.

Q25 Answer (b). Waves with short wavelength are less noticeably diffracted than waves with long wavelength.

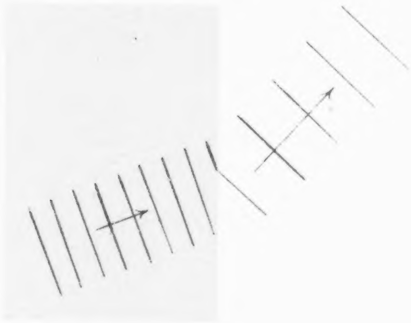
Q26 Because light diffracts, it is impossible actually to produce a "ray of light."

Q27 The angles are equal.

Q28 Parabolic.

Q29 The reflected wave fronts are parallel wave fronts.

Q30 1) Frequency does not change.
2) Wavelength decreases.
3) Its direction of propagation becomes closer to the perpendicular to the boundary between the media.



Q32 Superposition, reflection, refraction, diffraction, interferrence.

Q33 Sound waves are longitudinal; only transverse waves can be polarized.